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Time, Number, Ideal Genesis

(Chapter 8 of Draft MS: *The Logic of Being: Heidegger, Truth, and Time*)

I

In the 1968 article "Ousia and Gramme: Note on a Note From *Being and Time*", Jacques Derrida carries out a rigorous deconstructive reading of a footnote in the last chapter of division II of *Being and Time*. In the note, Heidegger asserts the direct connection of Hegel's conception of time to Aristotle's and the determination of both by the "ordinary" or "vulgar" conception of time as a "leveled off" series of present "now" moments, the concept of time which is, for Heidegger, characteristic of metaphysics in its privileging of presence in general. The reading yields terms in which Heidegger's assertion of this connection, and along with it his entire opposition of an "ordinary" temporality linked to metaphysics and to presence from the underlying "authentic" temporality of Dasein's ecstases, are put into question. In particular, by developing the implications of the originally aporeatic structure of Aristotle's discussion of time in *Physics IV*, Derrida can argue that the constitutive problems in terms of which time is thought by Aristotle remain characteristic of every subsequent discussion that recognizes time "as the condition for the possibility of the appearance of beings in (finite) experience" (p. 48) and thus, and even in exemplary fashion, for Heidegger's *own* discourse on time as well. Through the connection that links every discourse on time to the question of the conditions for the possibility of finite appearance, Derrida suggests, every such discourse remains characterized by a "profound metaphysical fidelity" to the thought of presence. This fidelity is marked most of all in those moments where time is subtracted from the realm of positive beings in order to appear as an underlying form of their appearance, of presentation or of presencing in general.

Such a moment, according to Derrida, is as much characteristic of Kant's conception of time as it is of Hegel's and Aristotle's; and it is once more characteristic of the determinative moment of *Being and Time* in which Heidegger *repeats* the critique of the "silent" determination of the nature of time by the assumed presence of some present being that already in fact characterizes the discussions of all three earlier philosophers. The insistence of this repetition is characteristic of a necessity that will have constrained the questioning of presence on the basis of time to remain in a certain sense "within" this metaphysics itself, repeating it by explicating its principles, or recovering what are its original paradoxes by putting them more radically into play. In particular, if Aristotle's discourse on time is irreducibly situated, Derrida suggests, within an interrelated series of aporias about time and the "now," aporias that are never resolved within Aristotle's text or indeed anywhere else in the history of metaphysics, the necessity of their repetition will have determined a certain necessary submission of the critical destruction of metaphysics on the basis of time to metaphysics itself. The "formal necessity" by which

every discourse of metaphysics carries with it both the resources of the "vulgar" concept of time and its deconstructive critique arises already, according to Derrida, as soon as the *sign* "time" begins to function in discourse and is an outcome of this functioning itself. It is thus an exigent task for deconstruction to examine the necessity of this repetition of the original aporias of being and time, Derrida suggests, and indeed even to "formalize" its "rule." (p. 48). As such a formalization of the necessity to repeat the original aporias, a necessity lodged in their own structure, this deconstruction would illuminate or formally indicate the original problem of the givenness to thought of the relationship of being and time as such. In this section and the next, I will attempt to follow this formal illumination of this problematic as it occurs in Derrida's text and Heidegger's, but also to formulate its original link to the problem of the givenness of number, which is laid bare in a radical way by the mathematical and metalogical thought of the twentieth century. In connection with some suggestions of the late Plato, I shall also suggest a basic link of this problem to the problem of the ideal genesis of number, in relation to the finite and infinite as such.

Aristotle's explicit discussion of time in the *Physics* begins by proposing to work out (*diaporesai*) two questions which, as Derrida points out, both gesture, by way of what Aristotle characterizes as an "exoteric" argument, to basic aporias of the constitution and nature of time. The first is the question whether time is a being or not (*ton onton estein e ton me onton*), and the second is the question of its *phusis*. The difficulties involved in both problems will lead to the opinion that time "does not exist at all or only barely, and in an obscure way." Most immediately, there is a problem about how time can exist at all, given that one part of it is no longer, and the other part is not yet. But both "infinite time" and "any time you like" are made up of these parts, each of which thus seems not to exist, and it is natural to conclude that something whose parts do not exist cannot take part in being (*metexein ousias*) at all. The discussion proceeds as a consideration of the nature of the "now" (nun), which appears to be the boundary between past and future, and its possibility. Is the "now" always the same, or is it continually or continuously "different and different"? The second hypothesis is untenable. For on it, if the moments succeed one another without interval, each new "now" moment will replace the last and the last will not, then, exist; or if there are moments between one "now" and the one that succeeds it then these intervallic moments, of which there are innumerably (*apeioros*) many, will be simultaneous, which is impossible. But the first hypothesis is equally so; for if the "now" is always the same, then both what is "before" and "after" would always be in this same "now" and "things which happened ten thousand years ago would be simultaneous [*hama*] with what has happened to–day, and nothing would be before or after anything else." These are the problems that will allow Aristotle to say that the "now" both that it is the "same", in one sense, and that it is not, in another, and that time is both a continuity with respect to the "now" and divided by it. (219b; 220a; Derrida p. 54). For this reason, he will apparently reject the claim that time is to be seen as composed of "nows" as a line may be thought to be composed of points; but this does not mean that he clearly or entirely rejects the idea of the "now" as a limit. Nevertheless, the sense in which the "now" is a limit between past and future is itself aporeatic: for a point to be a limit between two spans, it will have to be the end of one and the beginning of the other. For this to happen, the "now" will have to involve an "arrest or pause", but there is no such pause among the constantly flowing nows.

As Derrida suggests, the problems here posed are, in one sense, not distinct from the problems posed in general by the mathematical question of the relationship of the point to the line; but by the same structure and at the same time, they are none other than the problems of what jointly allows space and time to be thought at all. If the aporias of its constitution from "now" moments shows that time is not to be thought as composed of points at all and is in some sense irreducibly continuous, still it cannot be identified with the *gramme* as the linear inscription in space. For the spatially inscribed line is such as to have all of its parts co-existent *at once*; but it is of the essence of time, however it is composed, that its parts do not exist simultaneously. More generally, in thinking the difference between space and time in as a constituted and given difference, we think it exactly as the difference between the order of coexistence in the same time and the order of succession in which there is no possible coexistence in this sense. As Derrida points out, it is not even possible to say meaningfully that the coexistence of two "nows" is impossible, for the very sense of coexistence is constituted by this impossibility. Thus, "Not to be able to coexist with another (the same as itself), with another now, is not a predicate of the now, but its essence as presence." (p. 55). The very *meaning* of the present is constituted by this "impossibility", and thereby, Derrida suggests, so is "sense itself," insofar as it is linked to presence and *its* possibility. According to an aporia which is already implicit in Aristotle and is repeated in Hegel's discussion of time as the dialectical "solution" of the contradiction between the (spatial) point and the (spatial) line, the "with" of time (simultaneity) will thus presuppose the "with" of space that it also constitutes. If Aristotle is able to presuppose the difference between space and time as the difference between the order of coexistence and the order of succession, the supposition will be maintained only on the ground of a more basic structure of paradox which is at the same time evaded or dissimulated, as much in Hegel's explicitly dialectical discussion as in Aristotle's own. To assume the difference between space and time in this way is, Derrida suggests, to assume that it is already possible to know what it is to ask what time and space *are* in general; and thus to assume that one already knows that the question of essence can be "the formal horizon" of the question about both. But this is to assume that what essence *itself* "is" has not been "predetermined secretly – as presence, precisely – on the basis of a 'decision' concerning time *and* space." (p. 56).

The question is evaded in Aristotle by means, Derrida suggests, of his reliance on the resource of a single word which is according to its sense undecidable between a spatial and temporal significance, or rather whose sense is constituted by an undecidability between time and space. Aristotle can give himself the difference between time and space only on the basis of both *presupposing* and *foreclosing* this specific undecidability:

> Now, if Aristotle gives himself the difference between time and space (for example, in the distinction between *nun* and *stigme*) as a constituted difference, the enigmatic articulation of this difference is lodged in his text, hidden, sheltered, but operating within complicity, within the complicity of the same and the other, within the *with* or the together, with the *simul* in which Being-together is not a determination of Being, but the very production of Being. The entire weight of Aristotle's text comes down upon a word so small as to be hardly visible, and hardly visible because it appears self-evident, as discreet as that which goes without saying, a word that is self effacing, operating all the more effectively in that it evades thematic attention.

That which *goes* without saying, making discourse play itself out in its articulation, that which henceforth will constitute the pivot [cheville] (clavis) of metaphysics, the small key that both opens and closes the history of metaphysics in terms of what it puts at stake, the clavicle on which the conceptual decision of Aristotle bears down and is articulated, is the small word *hama*. It appears five times in 218a. In Greek *hama* means "together," "all at once," both together, "*at the same time*." This locution is first neither spatial nor temporal. The duplicity of the *simul* to which it refers does not yet reassemble, within itself, either points or nows, places or phases. It says the complicity, the common origin of time and space, appearing together [com-paraltrej as the condition for all appearing of Being. In a certain way it says the dyad as the minimum. But Aristotle does not say it. He develops his demonstration in the unnoticed self-evidence of what the locution *hama* says. He says it without saying it, lets it say itself, or rather it lets him say what he says.

By taking advantage in this way of the resource of the undecidable meaning of "hama", Derrida suggests, Aristotle can suspend his entire discourse, and with it the whole tradition of discussion of time and being that follows it, upon the original structure of aporia which it already involves. If this is correct, the original undecidability of *hama* points not only to, as Derrida says, the "small key that both opens and closes the history of metaphysics," but also to this originally paradoxical structure of time, a structure that also underlies the specific possibility of the critique of presence in general on the basis of time and which therefore *cannot* be closed or resolved by its means, but only (more or less explicitly) repeated.

To the extent that Aristotle himself is able to arrest or remove from play this original undecidability, it is by appealing to the distinction between potency and act, or between *dunamis* and *energeia*. Thus, as Derrida suggests, Aristotle can resist the claim that the *gramme* is as such a series of points, each of which amounts to a limit, by considering that the point *as limit* does not exist essentially and "in act", but only as potency and as accident. In particular, given this distinction, Aristotle can argue (220a) that just as the point, as boundary between two line segments, does not exist in actuality "in" the line but only in that and insofar as it is *drawn*, so the "now" as boundary is not an actual part of time but only potentially so; and thereby the necessity that the line be constituted by points is apparently avoided. There is thus no need to assume an actual "pause or arrest" of time in the now as boundary, but only the *possibility* of drawing a distinction between past and future in general at any time, and the problem of positing such a pause is apparently avoided. The argument is, further, facilitated by the relationship or analogy between time and motion that Aristotle develops as a consequence of the "most usual" supposition about time, that it is a kind of motion or change. Aristotle contrasts this with the views that time is the circular motion of the whole, or that it is this whole (as sphere) itself. The first is to be rejected, since a partial revolution would also take time, but would not be the revolution of the whole, and since there might be other heavens which would then have their own time; but the second is also to be rejected, since it leads to "too many impossibilities to be worthy of consideration". Aristotle thus proposes beginning again by considering the view that time, though it is not the motion of the whole, is some kind or type of motion. But this view, too, must be rejected. First, motion or change is only in the thing that moves or changes; but time is "equally present everywhere and with all things". Second,

motion or change is fast or slow, but time itself is not. Aristotle concludes that time is not motion, but it is nevertheless not independent of motion; indeed, Aristotle goes so far as to assert, if there were no motion, there would be no time.

He argues for this on the basis of the observation that we *sense* time *simultaneously with* motion: "it is together that we have the sensation of movement and time." (*hama gar kineseos aistheanometha kai khronou*). When we are in the dark and are not externally affected by any body, we may still perceive time; in this case, it is a movement that appears to be in the soul that is perceived or observed. It is thus that the perception of time is dependent on that of movement, and if (Aristotle concludes) it is only when we "perceive and distinguish" that time seems to elapse, it follows that time is, if not actually movement, at any rate something "belonging to" it.

This analogy or unity between motion and time allows Aristotle to argue, in the present context, that there is no aporia involved in the now as limit; for motion is as such continuous, and has a limit only in its possibly being completed or broken off. Analogously or for the same reason, the "now" which distinguishes before and after with respect to time is not its real constituent, but only a product of the potential distinction, which may be, but need not be, drawn at any point. In this way, Aristotle links the *actuality* of the "now" as limit to the activity of the mind's perceiving or distinguishing, an activity whose structure itself also verifies that time is something "belonging to" motion in the "simultaneity" or "togetherness" (hama) of the way both are given. Even when this co-givenness of time and motion does not involve the psuche's being affected from without, it is thus still on the basis of its own activity or its *seeming* to be active that time is originally given.

As Derrida notes, to understand the basis of time in this way is already to make it something like the form of inner sense. This is the form of a capacity to be affected in general, whose ultimate basis is the thought of the mind's self-affection in the interiority of its own self-presence. Aristotle has thus anticipated, even in detail, the structure of Kant's conception of time and indeed, just as much and with the same structure, the terms in which Heidegger will both repeat and criticize it in *Kant and the Problem of Metaphysics*. In particular, the analogy or connection that Aristotle already draws between motion and time thus both includes and dissimulates the original form of given time as a paradoxical auto-affection that is equally, and primordially, active in the giving and passive in the taking and in which the mind is both receptive in perception and active in creating its very possibility. If Aristotle can already pretend to resolve the aporia of the presence of the now by appealing to the distinction between the actuality of the continuous and the mere possibility of its discontinuous limit as drawn, he can therefore do so only by suppressing or evading the terms of this originally paradoxical structure of the givenness of presence, which will thus itself determine its own more or less critical repetition, in Kant, Hegel, and Heidegger himself. The form of this givenness can then only be determined, as Derrida suggests, as the finitude of a circle that "regenerates itself *indefinitely*", that constantly gives the possibility of the present without ever giving it as actual end. The very structure of the present is thought in terms of this auto-affective circle in each of the figures that interpret the possibility of time in terms of the possibility of a giving of presence to an intellect determined as finite. It is within this circle that all thinking of time as experienced or experienceable, as thought or thinkable, will then take place. Correlating the original

structure of the present to this paradoxical structure of circular self-givenness, it will thus also pre-determine the constitutive terms of critique in which any reflexive consideration of the privilege of the present can subsequently take place.

But there is another essential component to Aristotle's analysis of the relationship between time and motion, one that equally and just as essentially determines its structure as given in Physics IV and also (as I shall argue) the possibilities of its "metaphysical" repetition, though it also (as I shall try to show) provides for a certain kind of problematic communication of the "metaphysical" discourse on presence and the limit with something that is no longer simply metaphysics, or (more accurately) never was, but nevertheless itself determines the problematic basis of the finite and the infinite *as such* in more original terms. This further component is the thought of *number*; and the more original problematic discourse of the finite and the infinite is mathematics itself. For while on the one hand the constitutive relations of continuity and discontinuity, or of the finite and the infinite, are unfolded and developed as posed problems (and were already in Aristotle's time) through the actual practice of mathematicians, on the other hand what is thereby unfolded provides terms which illuminate the very possibility of the givenness of the infinite as such. In construing time in relation to motion as its number, Aristotle is doubtless already aware of a peculiar relationship between time and the mathematical infinite, one whose resources he in fact relies on in detail, as I shall try to show, though he also dissimulates and represses it, by means of the distinction between the potential and the actual, and by means of his official view that mathematics is, as such, only a *regional* discourse. But this relationship can be brought to light in a way that both verifies and displaces the deconstructive analysis, as I shall try to show, by developing the consequences of the formal thinking of formalism itself, especially when it is a thinking by means of formalism of its own inherent limits. Such a thinking, in which what is at stake is no longer the limit provided by any empirical figure of restriction or any pre-figuration of the actual as dunamis, is rather illustrative of the very constitution of the limited, and hence of what can be held within the limits of any presence. As such, it is an original thinking of the limited and unlimited that was already actively pursued in Plato and Aristotle's time, particularly under the impetus of the discovery of irrational magnitudes such as the diagonal of the square. But it is transformed and put on a new footing through the radical discoveries of the twentieth century in its reflexive investigation of formalism and its own limits, whereby it can illuminate in an original way, as I shall try to show, the "ontological" problematic of the givenness of time. The structure here is nothing like that of an unrestricted mathematicism, or a simple and direct application of calculative or mathematical thinking or structure to a matter given in itself prior to that application.   It is rather that of what may be seen as the internal dialectic of limit and unlimited which structurally unfolds the basis of any such "application", or which does not precede its actuality as a condition for possibility but rather inheres in its actuality as its *real* virtual structure.   As Lautman himself suggests, we may see the reflective discoveries of twentieth-century mathematics, especially in those of its areas that are usually described as the theory of computation and the theory of proof, as themselves providing an original and more basic illumination of the underlying structure of paradox in which, according to Derrida's argument, all metaphysical thinking of time and presence, including Heidegger's own, is lodged. If this is correct, the analysis will then also deepen in an obvious way the "ontological" problematic of the givenness of time, on which Heidegger himself formulates the general critique of what he treats as the metaphysics of presence.

How, then, does number enter into Aristotle's analysis of the structure of time? At first glance, secondarily and by analogy, although it will also basically determine what is Aristotle's most official "definition" of time: that it is "the number of motion with respect to before and after." In particular, having introduced the problematic "non-independence" of time from motion in virtue of which it is possible to consider time as something "belonging to" movement, Aristotle can consider the apprehension of time as analogous to or actually dependent on the marking of a movement by the judgment of a difference of "before" and "after" between two of its points. With the marking comes the possibility of measuring the interval as such, and of considering what is so bounded; when we consider the interval as bounded by two distinct "nows", Aristotle suggests, we consider it to be time. If we perceive a "now" alone as one, we do not perceive its relation either to another moment or to a "before" or "after"; then we do not, according to Aristotle, perceive time. On the other hand "when we do perceive a 'before' and an 'after', then we say that there is time." And this leads to the "official" definition: "For time is just this–number of motion in respect of 'before' and 'after'."

The definition depends on a twofold applicability of number, both to magnitude in general and to motion, and an analogy between the two kinds of application: since "we discriminate the more or the less by number, but more or less movement by time", time is *itself* a kind of number. What is intended here is, Aristotle clarifies, not that time is a "numbering number", the kind of number with which one counts, but rather a "numbered number" , the kind of number that is counted. Time is number in the sense in which one says "the number of horses" or "the number of men", which may be the same, whereas the things concerned are themselves different. Time is not, then, a mathematical being such as a number in itself; rather it is what is counted in the counting, or what is discerned in the discrimination of "before and after." It is in *this* way – and only this – that the "now" itself exists; it "corresponds" Aristotle says, to a "body that is carried along" in motion in that we become conscious of the "before and after" with respect to it, but it only when we regard these as counted or countable that we get the "now" itself. Time is not motion, but it is motion "insofar as it admits of enumeration." ()

As Derrida notes, the categories that govern the whole relationship of number, motion, and time, are those of *analogy* and *correspondence* (p. 58). Time is on the one hand analogous to motion, in that what one can say of motion (its continuity, its structure of the "before and after") one can say of time as well; on the other hand, it "corresponds" to motion, as in the correspondence of the "now" to the body that is carried along *spatially* in a motion, or it actually *is* motion, "insofar as" it *can* be enumerated. Again, time is the counted *number of* motion (and in that sense corresponds to it as number corresponds to what is numbered, for instance as "10" corresponds to the group of 10 horses) or it is what *is* counted in the counting, as (analogously) motion is itself measured in determining two distinct points within a unitary motion as boundaries of a span. If Aristotle can alternate between the relations of correspondence, analogy, and identity with respect to motion and time in this way, it is because the whole construction turns on the dependence of time on the awareness that measures it, and thereby on, as Derrida shows, the ultimate simultaneity *or* togetherness (*hama*) of the perception of both. By privileging this *hama*, Aristotle can foreclose the aporias of the actually existing "now", treating it instead as the merely *potential* outcome of an act of measurement, an accident with respect to time in

7

itself, rather than its actually constituting substratum. But this is itself only possible insofar as the essential undecidability of the *hama* remains, and is accorded an absolute privilege as the very form in which the whole considertation takes place.

On the terms of analysis that Derrida suggests, Aristotle will have presupposed or actually performed a derivation of the structure of time from the conditions of its givenness as the possibility of its perception by a finite being and from its possible measurement *by this being*. Hence, he will have derived it from the present being of a presence in general whose form is finitude in general. Only a being that is in general in the situation of having to perceive motion and change outside it, and is in particular in the position of being able to perceive in a similar fashion its own internal motion, will be able to count time in the way that Aristotle considers definitive of it. The structure is thus one of a kind of criteriological derivation of the structure of time *from* what appears to be the necessary form of its own givenness to a finite being; this is the structure that, as Derrida emphasizes, will be repeated, more or less explicitly, in Kant, Hegel, and Heidegger, and will also, in each case, in each case and by the same structure, conversely suggest terms for the critique of the assumption that time is silently determined simply *by* the being to whom it is given.   There is no simple escape from this circle, as Derrida says, because it is characteristic (as he also says) of the underlying logic of the signification of "time" – as soon as it is a signifier that can enter language – at all. As such, it propagates its structure on each discourse and each analysis that attempts to make the structure of time evident by considering the form of its givenness

However, without breaking out of the circle or declaring it at an end, it is nevertheless possible to reconsider its structure in relation to a constitutive topic that is, doubtless, presupposed as determined in every one of these instances and as determining with respect to each of these structures, but is itself not simply a chapter of the "metaphysics of presence" or of metaphysics in general. This is the topic of the *relationship* between the finite and the infinite and the *availability* of the infinite as such. This topic, as it has been pursued in the actual discourse and discoveries of mathematicians since the Greeks, is not primarily or generally undertaken on the basis of an idea of the "constitutive finitude" of the subject, human, self, or person, but it has obvious *relevance* for any conception of time that is criteriologist in the sense that Aristotle's is: that is, for any conception that relates time essentially and constitutively to the possibility of its counting and measuring. For if the conceptions of Aristotle, Kant, Hegel, and Heidegger himself *each* recurrently reformulate, as Derrida suggests, the circle of the criteriologist definition of time as givenness to finitude and its critical undermining by means of the question of the basis of its giving, each also presupposes, in presupposing the very possibility of *counting* time as such, the *availability* of number, as such and "in general" for the counting. To presuppose this is in itself to presuppose one determinate conception of the relationship between the finite and the infinite or another, and the necessity of the presupposition effectively opens the question of the constitution of time to the effects of the investigation of the infinite that is, in many ways, constitutive of mathematics as such. As we have already suggested (chapter 5 above), the ultimate consequences of this investigation in twentieth century mathematics and logic, in unfolding and decomposing the structural idea of an effective procedure on which the concept of a regular "progression to the infinite" rests, are such as to call into question the very ideas of completeness and consistency with respect to the constituted abilities of a finite agent.   They thereby suggest a certain kind of displacement of any

conception on which the capacities of such an agent are constitutive of the infinite as such, or (seen another way) point to its insistence, in what Godel called the "inexhaustibility" of mathematics, beyond anything that can be simply ascribed to the forms of its givenness to such an agent. This is not, in itself, to break the circle that recurrently links the "metaphysics of presence" to its own internal critique on the basis of given time; but rather to resituate it on rather different ground which also (as we shall see) puts it in relation to a wholly different "text."

In particular: if Aristotle can, then, define time as the counted number of motion on the basis of the criterion of the soul as the agent of counting, he is only able to do so by assuming the availability of number *to anyone* and *in general* of the numbers that count. The implications of the assumption are particularly evident when Aristotle considers the objection that, if time is dependent on counting, without the soul there would be no time:

> Whether if soul did not exist time would exist or not, is a question that may fairly be asked; for if there cannot be some one to count there cannot be anything that can be counted, so that evidently there cannot be number; for number is either what has been, or what can be, counted. But if nothing but soul, or in soul reason, is qualified to count, there would not be time unless there were soul, but only that of which time is an attribute, i.e. if movement can exist without soul, and the before and after are attributes of movement, and time is these qua numerable.

> One might also raise the question what sort of movement time is the number of. Must we not say 'of any kind'? For things both come into being in time and pass away, and grow, and are altered in time, and are moved locally; thus it is of each movement qua movement that time is the number. And so it is simply the number of continuous movement, not of any particular kind of it.

> But other things as well may have been moved now, and there would be a number of each of the two movements. Is there another time, then, and will there be two equal times at once? Surely not. For a time that is both equal and simultaneous is one and the same time, and even those that are not simultaneous are one in kind; for if there were dogs, and horses, and seven of each, it would be the same number. So, too, movements that have simultaneous limits have the same time, yet the one may in fact be fast and the other not, and one may be locomotion and the other alteration; still the time of the two changes is the same if their number also is equal and simultaneous; and for this reason, while the movements are different and separate, the time is everywhere the same, because the number of equal and simultaneous movements is everywhere one and the same.

> Now there is such a thing as locomotion, and in locomotion there is included circular movement, and everything is measured by some one thing homogeneous with it, units by a unit, horses by a horse, and similarly times by some definite time, and, as we said, time is measured by motion as well as motion by time (this being so because by a motion definite in time the quantity both of

9

the motion and of the time is measured): if, then, what is first is the measure of everything homogeneous with it, regular circular motion is above all else the measure, because the number of this is the best known.

It is notable that Aristotle considers the objection, but also that he does not answer it straightforwardly. He is willing on the one hand to endorse the claim that time is dependent on the soul, but tries, on the other, to limit the consequences of this by appealing to the possibility of coordinating the counting in such a way that the individuality and the temporal limitedness of the soul does not matter. If time is the counted number in, or of, a motion, it will be present only when, and insofar as, there is a counter. This would seem to imply that there could be many times, corresponding to the various souls or motions, and that time does not indeed have the character of being "in everything, both in earth and sea and in heaven". But both conclusions can be blocked, on Aristotle's argument, if it is possible to *coordinate* the measurements of the individual motions by means of a standard that is regular and uniform. If two spans that are simultaneous (hama) can also be "equal" in that they begin and end at the same moments, they are not two simultaneous times but the same one. But even if two times are not simultaneous and are thus different they can be equal by being the same "length" of time. The identity is akin to the identity of the number 7 in the groups of seven dogs and seven horses; the groupings are of different things, but there is nevertheless something in common in their measure. In both cases (extending the metaphor) the measure depends on the particular unit; thus, as groupings of horses must be "measured" by the horse, so time must be measured by something "homogenous" with it. This something is the regularity of a circular motion, which functions as a standard for the counting of time that is everywhere and to everyone accessible (not accidentally, it is the "best known" of motions). The regularity of the motion implies the possibility of using it in general to measure time, which possibility it both creates and regularizes: the regular motion of the heavenly bodies serves as a standard that is accessible to everyone living "under the sun," watches and clocks are to be synchronized. It is implicit in this that the regular standard is as such repeatable, anywhere and at any time, and is moreover accessible to anyone in general. Aristotle finds the form of this standardization in circular locomotion, which is structurally and identically *iterable*, capable as such and by the simplicity of its form of being repeated anywhere in general, and at any time.

In thus appealing to the possibility of a general structural iterability, Aristotle limits or modifies the consequences of the dependence of time on the soul's activity of measurement by submitting it to another condition that is also implicit in the activity of measurement in general, that of the general availability of the *standard* and its repeatability *ad libetum*. Like the availability of number for counting, to which Aristotle compares it, this availability is *in principle* unlimited: it is only if one can assume that the standard is always available, and everywhere, that it will be usable at all; only in this way will it be possible to vindicate the claim, which is surely correct, that time is not only or just "in the soul" but is *also* "in everything, both in earth and sea and in heaven". In appealing to the standard or using it, one applies in a particular case a structure that is in itself self-similar across *all* the cases of its particular application and is always and in general applicable. Both the (unlimited) *potentiality* of this possibility of application and its *unlimitedness* in principle determine equally its structure: even if the standard is not actually applied everywhere and all times, it must be possible to do so, and this possibility must never

give out. The appeal to the standard which blocks or modifies the problematic implications of Aristotle's identification of time with the soul's particular activity of measuring is thus inherently and structurally related to the specific structure of the infinite, here under the particular conception of the *indefinitely* repeatable as such. As such, if Aristotle can avoid the further consequences of saying that time is simply motion or what is measured in the measuring of it, it is because he can appeal to the relationship, both identical and metaphorical, of this application of the standard to the use of number in counting, and thereby to the (metaphorical or actual) identity of this availability with that of number. In terms of this analogy or identity, it is crucial that number is, as such and in itself, iterable in two senses: both in the indifferent availability of one and the same number, say 7, to serve for the measure of distinct groups of different kinds of things, and in the indefinite possibility of generating numbers themselves by iterating the "plus one".   In both senses, the standard itself is determined as indefinitely iterable, everywhere and in general, and this indefinite iterability is essential to the very structure of counting as such that is not only criterial for time, according to Aristotle, but generally definitive of it.

There is good reason to think that this indefinite accessibility of the standard, as the possibility of counting, is itself determined, in Aristotle's discourse, by a particular conception of the infinite and *its* availability to thought. To begin with, the account of time given in *Physics*, book 4, is both preceded by and visibly prepared by the discussion of the infinite in book 3. Over the course of this discussion, Aristotle argues that it is not possible for any actually completed infinite magnitude to exist and hence, as a consequence, that no actual material object can be infinite in size. This is because the infinite by addition exists always only potentially and never actually; what is infinite in this sense has the character of "always" being able to be added to but is never an actually existing infinite in the sense in which a sculpture exists as complete and actual. This does not preclude, however, that continuous magnitudes are divisible *in infinitum*; indeed, Aristotle suggests in introducing the topic of the infinite, the specific character of the infinite is first and most directly shown in connection with the continuous. Nor is it to say, however, that there is not the infinite at all and in some sense; indeed, Aristotle lists five considerations that point to its existence, and to the "many absurdities" that would result if it did not. The fifth and most telling of these is the consideration that "not only number but also mathematical magnitudes and what is outside the heaven are supposed to be infinite because they never give out in our thought."

This is related to the other considerations in favor of the infinite that Aristotle introduces: that the limited always finds its limit in something else, that "coming to be and passing away do not give out," that magnitudes are infinitely divisible in a mathematical sense, and indeed to the consideration that he places first, that time, in its phusis, is infinite. Aristotle never disputes this claim, either in book 3 or in book 4; nor does he challenge the structurally determining relationship he points to here between this infinitude of time and the character of numbers such that they "never give out" in thought. Rather, his strategy is to *reinterpret* this determinative "never giving out", both of number and of time, in terms of the distinction between potentiality and actuality:

> The infinite exhibits itself in different ways–in time, in the generations of man, and in the
> division of magnitudes. For generally the infinite has this mode of existence: one thing is always

being taken after another, and each thing that is taken is always finite, but always different. Again, 'being' has more than one sense, so that we must not regard the infinite as a 'this', such as a man or a horse, but must suppose it to exist in the sense in which we speak of the day or the games as existing things whose being has not come to them like that of a substance, but consists in a process of coming to be or passing away; definite if you like at each stage, yet always different. But when this takes place in spatial magnitudes, what is taken persists, while in the succession of time and of men it takes place by the passing away of these in such a way that the source of supply never gives out.

The characteristic of "never" giving out that is characteristic of both number as thought and time as counted is thus interpreted, not as pointing to the source of both in some principle or basis of plenitude which underlies it, but rather as the boundlessness of a potentiality that is never fully exhausted in the completeness of its actualization. This is the potentiality of what, in its taking, "always" involves takes something "outside" itself. In the taking, what is taken is, as such, finite. But it can always again be taken, and the taking is in each case "always different". The "always" that is applicable here to magnitude as such is not applicable in the same way to things that may exist fully and actually, such as bodies, whose being comes to them "like that of a substance." Nevertheless, it is in a certain way the specific formal basis of potentiality as such, for bodies and substances that can exist in full actuality just as much as for *taken* processes and magnitudes for which, as Aristotle says, the "source of supply" of the possibility of taking "never gives out." For even in the case of fully actual beings, their potentiality precedes their actuality as the principle of *its* coming-to-be; the transition from potentiality to actuality is the form of the coming-to-be and, in this way, the procedural or temporal *basis* of determinate being. Here, as Aristotle elsewhere suggests, potentiality is opposed to actuality as matter is opposed to form; the subsistence of matter in form amounts, on the one hand, to the determining possibility of its coming to be actual and, on the other, to the substrate of its *actual* being, its determinate being thus-and-so and thereby its being measurable, as finitely determined and within always finite limits.

In the measurement of finite things, they are determined to be thus and so; for example, a distance is determined as having some finite extent, or a span of time between two events is determined as having some length. What is determined in the determination cannot exceed the finite, even if the possibility of determination itself always goes further than any finite limit. This is what allows Aristotle to argue that the potentiality divisibility of magnitudes and times in *infinitum* does not imply the actual existence, as underlying stratum, of any infinitely determined point; and in this way to resolve or foreclose the aporias of the actual constitution of the continuous from the discontinuous. But if the idea of potentiality can serve Aristotle, in this doubled fashion, as both the principle of coming-to-be of limited things and the basis of the *unlimited* possibility of their measurement as being thus-and-so, there nevertheless still arise certain formal antinomies characteristic of the assumed character of the unlimited possibility of measurement itself, and in particular the character of its *always* being possible, or its *never* giving out.

What, in particular, is the form of this potentiality: what guarantees the "always" of the unlimited possibility of continuation, and the "never" of the "never giving out", both in the case of number as thought and time as counted? How does one know, in general, that the possibility of counting will

"always" continue, that the "source of supply" will "never" subside? Assuming the terms of the analysis, this question does not simply involve the structure of counted time, for it applies just as much to the measured magnitude of spatial and other continuous quantities. Nevertheless it is "temporal" in either case, and even in a pre-eminent sense. For what is here thought as the general form of an unlimited potentiality of "taking" in measurement is thought, in either case, as the unlimited potentiality of the *serial* continuation of an iteration, the endless possibility of repeating what is in some way the "same". In the passage just quoted, Aristotle describes the mode of existence of the infinite as that in which "one thing is always being taken *after* another" and indeed in such a way that what is taken is "always finite, but always different". Both the "after" and the "always" are, here, irreducibly temporal, as is (indeed), as we have seen in connection with Aristotle's use of the hama, the basis of the assumption that what is taken is as such always finite. But if Aristotle can here claim that the taking is *of* what is again and again different, the possibility of the taking is in general guaranteed, as we have already seen in connection with the specific analysis of time as the number of motion, only by the subsistence or availability of what is again and again the same.

The possibility of the iterative "taking" of difference, to which Aristotle here points as the basic character of the infinite, is thus itself dependent on a more basic repetition of the self-identical same, whose irreducibly spatial/temporal figure Aristotle finds, as we have seen, in the repetition of circular motion. By means of the same argument, the possibility of the infinite to comprehend the different in general is thus subsumed, or subtended, under a more structurally basic principle of identity, the repetition of the self-identical in its regular series. With this, the tendency of becoming to produce or induce difference is itself controlled, placed under the assurance of a regularity, constrained within the boundaries of a limit that guarantees that it can always be measured. This is already the structure of the measurement of spatial distance, which presupposes the serial applicability of a standard that measures the unit, as when a meterstick is applied repeatedly to the span to measure it. Here, as Aristotle says, measurement is the measurement of what remains in coexistence when the measurement is taken; even the spatial case is thereby essentially determined, as Derrida points out, by the actually irreducibly spatial *and* temporal category of coexistence. But the case of the actual "measurement" of the span of time between two moments, where what is continuously or repeatedly taken in the measuring, as Aristotle says, does not continue to exist in the present but rather is such as to "pass away," brings out the underlying aporia of identity, repetition and difference in yet a clearer and more direct way. This aporia is, as Derrida suggests, nothing other than the aporia of the existence of the "now" in its constant "coming to be" and "flowing away," and as such defines the multiply aporeatic structure of the presence of the present as such.

What consequences follow, if these aporias are not arrested, foreclosed, or avoided, as they are in Aristotle's text, but rather brought to light and formalized? The formal *problematic* that leads to them is then visible as a more general structure underlying them in the structural relationships of the ideas of the finite, the infinite, repetition, identity and difference, continuity and discontinuity as these are unfolded in the mathematical thinking of number itself. Here, the specific idea of the infinite as always only potential, on which turns Aristotle's whole analysis, is recognizable as grounded in *one* particular conception of the givenness of number; but it is not the only one, and alternatives are to hand in the

mathematical or metamathematical unfolding of the problematic relationships of these ideas in concrete thinking about the mathematical finite and infinite that articulate the paradoxes and point to their more original structure. What is more, just as Aristotle's specific conception of the infinite as potential determines the temporality of what is for him the being of becoming as well as the temporal form in which the being of the finite becomes known in measurement, each of these alternatives will suggest alternative images of time, each one articulated according to the paradoxical structures of limitedness and unlimitedness that insist in them, though differently in each case. Because of this insistence of the paradoxes of the finite and the infinite in each case, it will no longer be possible to think any of the structures involved as both wholly consist within themselves and complete in the sense of offering a unified and coherent schema for the application of mathematical thinking to the structure of time; rather, the structural question of the *applicability* of number to time becomes integral to the *topic* of the mathematical or metamathematical reflection on these constitutive ideas.

Methodologically, this has a consequence that is in fact already suggested by Derrida in his analysis of Aristotle: number or the mathematical in general can no longer be presupposed as simply exterior to the being of time, or opposed to it in the way that Aristotle does, as the counting number to the counted number, or as the determining is opposed to what is thereby determined. Rather, since the general possibility of the "unlimited" application of number to the determining or thinking of time determined as more or less itself becomes a topic for mathematical reflection on the finite and the infinite as such, the topic of the being of time can no longer be excluded from the proper scope of this reflection as accident is excluded from essence or as matter is opposed to form.

Henceforth, it will be of the *essence* of time that it be counted, or at least that it be determinately and originally related to number in *its* original givenness, and not simply as what is to be determined is related to what determines it. Without reducing it to "being" simply a mathematical object, it will then be possible to affirm that time is, at any rate, not *simply* extra-mathematical; at any rate it is not extra-mathematical in the sense in which horses or dogs, for instance, are extra-mathematical, even though, as Aristotle points out, the numbers of their groupings may be counted and compared.[1] But by the same

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[1] Cf. Derrida: "[For Aristotle] time is a numbered number…This means, paradoxically, that even if time comes under the rubric of mathematics or arithmetic, it is not in itself, in its nature, a mathematical being. It is as foreign to number itself, to the numbering number, as horses and men are different form the numbers that count them, and different from each other. *And different from each other*, which leaves us free to think that time is not a being among others, among men and horses." (p. 58-59). According to Derrida, moreover, this exclusion of time from the *being* of the mathematical is possible only on the basis of the argument, ambiguously itself both mathematical and non-mathematical, by means of which the "now" is treated as accidental and only potential with respect to the continuity of time in itself: "Like the point in relation to the line, the now, if it is considered as limit (peras) is *accidental* in relation to time. It is not time, but time's accident … The now (*Gegenwart*), the present, therefore, does not define the essence of time. Time is not thought on the basis of the now. It is for this reason that the mathematization of time *has limits*. Let us take this in all possible senses. It is in the extent to which time requires *limits*, nows analogous to points, and in the exent to which the limits are always accidents and potentialities, that time cannot be made perfectly mathematical, that time's mathematization has limits, and remains, as concerns its essence, accidental. A rigorously Hegelian proposition: let us recall the difference between the present and the now." (p. 61). According to Derrida, the mutual exteriority of mathematics to time, and hence the claim of the inherent limit of possible mathematization with respect to , will thus have been determined, in a way continuous

token and for the same reason, it will no longer be possible to exclude the mathematical in *general* from the "topic" of time. If this exclusion, whereby the mathematical as such has been maintained as separated from all possibilities of becoming and as the extra-temporal in itself, though it remains in an obvious way determinative of them, remains determinative for metaphysics as such, it is here possible to begin to grasp the possibility of an overturning or reversal of it within the ambit of a retrospectively more basic thinking of the being of finitude and the infinite themselves. Here, moreover, the constitutive figure of the infinite in its relation to finite time is not thought, in the characteristic mode of ontotheology, as an infinite-absolute, austerely removed from becoming and change. Rather, it is to be unfolded in the specific logical and metalogical structures that are indicated in the inherent paradoxes of mathematical being and its specific relation to finitude. With this, the characteristic discourse of the *phusis* or metaphysics of time, which Derrida suggests is structurally continuous from Aristotle to Heidegger, is made to communicate integrally with another kind of text, the text of mathematical reflection, or of a mathematical dialectic which is presupposed in every concrete application of the concept of number in counting time but is not itself simply "metaphysical" in this sense. The implications of this mathematical or formal text thereby also become relevant, in a direct way, to the "ontological" problematic of the original relationship of being and time, and the internal or external possibilities it structurally poses for the reversal of the "metaphysical" determination of this relationship are thereby more originally shown.

Even though it may seem to be presumptively and officially excluded in this context by Heidegger himself, the relevance of such a development of the mathematical and metamathematical problematic for the ontological relationship of being and time could be verified in many ways in reference to Heidegger's texts.   Here, I will just point to one place in which it is obviously relevant: Heidegger's most extended discussion of Aristotle's "treatise" on time, in *The Basic Problems of Phenomenology*. Heidegger here reads Aristotle as drawing out, with his concept of time, the determinate consequences of a specific interpretation of what it is to be *in* time: namely, that it is to be an object of nature, of the sort that is shown by our "natural" experience of things and of time itself. This is what, according to Heidegger, determines that he will privilege the character of local motion as the basis for his analysis of the structure of time in itself; for it is in such motion that time indeed most naturally and basically measured and experienced. Nevertheless, equally and at the same time, Aristotle holds that time is "in the soul", and this raises the *question* of how it can indeed *also* be everywhere and in all things. The question is particularly insistent, Heidegger notes, at the point at which, in concluding the whole discussion, Aristotle poses the question about whether time, as counting, would or could still exist without the counter. According to Heidegger, he does not resolve this question but merely "touches on it"; nevertheless it points, in the ontological context of Heidegger's own inquiry, to the further question of "*how* time itself exists." And this question, Heidegger argues, is not to be settled on the basis of any determination of time as "subjective" in belonging to the subject or as "objective" in being basically

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from Aristotle to Hegel, by the same thought that excludes the present from the flow of time by thinking it as its limit. In fact, in light of the present analysis, it is probably possible to draw out the connection even further, and to question on its basis the specific way in which the consequences of formalism, or of mathematization in an extended sense, are *standardly* put out of commission, arrested or ignored in considerations of the structure of time, not only from Aristotle to Kant to Hegel but also in Heidegger and even (it must be said) in Derrida himself.

determined by number. Rather, though an "unending dialectic can be developed here", no progress is made unless we get clearer about "*how* the Dasein's being itself is," and in particular how it is connected to the specific phenomenon of world. Because of and in this connection, "everything extant that the Dasein encounters is necessarily intraworldly, held-around [con-tained] by the world." (p. 255). In particular, to resolve the specific aporia of time *as the counted,* it is necessary to attend to the way in which Dasein's original being is already being-in-the-world, and so that conditions for the possibility of Dasein's being affected by objects in general, such that they can be measured in local motion, are themselves rooted in its structural transcendence, its openness toward the world as such. Nevertheless, Aristotle's indication of the numerical character of time is here treated as decisive, and even as something to be preserved in the course of the analysis that goes toward the more fundamental structure of underlying temporality:

> The numerical character of the now and of time in general is essential for the fundamental understanding of time because only from this does what we call intratemporality become intelligible. This means that every being is in time. Aristotle interprets "being in time" as being measured by time. Time itself can be measured only because on its part it is something counted and, as this counted thing, it can count itself again, count in the sense of measuring, of the gathering together of a specific so-many.

> At the same time the numerical character of time entails the peculiarity that it embraces or contains the beings that are in it, that with reference to objects it is in a certain way more objective than they are themselves. From this there arose the question about the being of time and its connection with the soul. The assignment of time to the soul, which occurs in Aristotle and then in a much more emphatic sense in Augustine, so as always thereafter to make itself conspicuous over and over again in the discussion of the traditional concept of time, led to the problem how far time is objective and how far subjective. We have seen that the question not only cannot be decided but cannot even be put in that way, since both these concepts "object" and "subject" are questionable…It will turn out that this manner of putting the question is impossible but that both answers – time is objective and time is subjective – get their own right in a certain way from the original concept of temporality. (p. 256)

After this, there follows an analysis of the derivation of the "natural" and "common" understanding of time, as it is evident in the use of a clock to measure time, from the more "original" and underlying structure of Dasein's own temporality. This is essentially an extended version of the analysis of the derivation of "world time" from the original ecstatic-horizonal temporality of Dasein that is given in *Being and Time*.

The analysis will lead Heidegger to suggest the prior rooting of the possibility of the kind of measuring of time that occurs in clock-reading and clock-using in the original temporality of the ecstases, by which Dasein gives itself and takes time for itself; in this original temporality, Heidegger suggests, we move constantly in a "silent discourse: now, not until, in former times, finally, at the time that, before that, and so forth." (p. 259). The derivation of the unlimited possibility of measurement that is characteristic of clock-time and world-time as so determined is therefore possible only on the basis of a levelling off or

privation of the basic character of significance, in terms of which, as Heidegger argues in Being and Time, time is always basically understood, in relation to Dasein's projects, as "time to…" Nevertheless, in locating the possibility of measured clock time in the prior structure of Dasein's temporality in this way, Heidegger nevertheless neither doubts nor challenges the universality of the *applicability* of time measured, on its own level, to *all* beings in general. This is the universality that shows up in Aristotle's development of the "numerical character of the now and of time in general", and it is from *this* character, and it alone, according to Heidegger, that the intratemporality of beings in general becomes intelligible. The generality of this intratemporality is not to be denied: indeed, every being is in time. And even if Aristotle's particular determination of it in terms of the possibility of the measuring of local motion causes the aporeatic question of its "subjective or objective" nature to arise in a particularly sharp form, the general form of the analysis, as an analysis in terms of the form of access to the phenomenon which is itself taken to prescribe the definition of the phenomenon itself, is not in question. It is only, here, to be deepened by undertaking a more ontologically basic analysis of this underlying form of givenness itself.[2]

As we have already seen in chapter 6, there are questions to be raised about the coherence of the "derivation" of world-time from fundamental temporality, insofar as this fundamental temporality is specified or specifiable as the *individual* Dasein of authenticity and projects whose fundamental and individuating horizon is death. Whereas, on the one hand, it is mysterious how any collection or colligation of such individual projects could *itself* yield the general and intersubjective time of the standard applicable to every such Dasein "simultaneously", on the other it is not obvious on the basis of this conception *how* the in-principle infinity of countable time is projectable or coherently applicable to events and phenomena taking place before the existence of any individual at all, or as they would be if no Dasein existed. As was suggested there, these problems can be ameliorated or even in large part resolved if we may take "Dasein" to be, in the relevant sense, not an individual at all, but rather an inherent structure of disclosure or of its formal-structural conditions, the inherent structural configuration of the interlinked possibility of sense and truth that Heidegger later thinks as the "open", which is not any longer the (individual) Dasein but rather its structural form and underlying condition.[3] As we are now in a position to see, however, this original structure, as much mathematical, cosmological, or logical as it is "ontological" in the sense of the "fundamental ontology" of Being and Time, also stands in an original and *basic* relationship to the structure of number and its givenness as such. This relationship, though it is not developed explicitly by Heidegger in the course of his analysis of the ontological problematic, and though it draws upon and unfolds the ideas of the finite and the

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[2] "…the Aristotelian definition of time does not contain a tautology within itself, but instead Aristotle speaks from the very constraint of the matter itself. Aristotle's definition of time is not in any respect a definition in the academic sense. It characterizes time by defining how what we call time becomes *accessible*. It is an *access definition* or *access characterization*. The type of definiendum is determined by the manner of the sole possible access to it: the counting perception of motion as motion is at the same time the perception of what is counted in time." (pp. 256-257).

[3] Because of Heidegger's later overcoming of the structure of transcendence (see chapter 4 above), it is also no longer a *condition of possibility* for the individual Dasein. It is not opposed to the "actual" Dasein as potentiality is opposed to actuality in general, but rather really inherent in it as a structural-ontological moment, more or less in the sense of what Deleuze will call the "virtual". (For more discussion of this kind of inherence, see chapter 9).

infinite in a way wholly different from the way that they are developed in *Being and Time* itself,[4] is also not opposed to anything suggested in that analysis, particularly if we can take the analysis of Dasein there already to gesture, in a certain sense, in its direction.

In particular, with respect to the analysis of the "basis" of world-time in Dasein's ecstases that is offered there, the problematic of the finite and the infinite as it figures in the very basis of the possibility of numbering and counting is an *equally* "primordial" one, articulated along a different dimension but just as "foundationally" significant to the analysis of Dasein's structural relationship to the possible givenness of sense and time.   Sense is projected, and the agent of the projection is (in some sense) Dasein, but the individual Dasein is neither the creator nor the controller of sense. Rather, in speaking or understanding a language I am always already in relation to possibilities that are not simply my own, and which I understand as pre-existing me and indeed capable of continuance beyond my own death. Sense is thus *real* in that it inheres in beings, in themselves and as they are in themselves; and this is equally (or eminently) the case, as well, for their mathematical structure and the temporal possibility of their repetition or continuance beyond the specific conditions of my own life and death. And similarly for time itself: even if my time and the horizon of my possibilities is set by the limitative conditions of my birth and death, it does not follow that I do not understand the time of my life as situated within the broader structure of time that pre-exists me and will continue after me. These fundamental structural possibilities do not need to be understood as corresponding to an ontic being that exists eternally, or to a place of eternal existence structurally subtracted *from* time, in order to be comprehended in terms of their more basic formal structure. Rather, they are illuminated, as I have suggested, in an irreducibly twofold way, both by the analysis of the underlying structure of the individual Dasein which is "in each case mine" and by the anonymous conditions revealed in the internal mathematical consideration of the problematic of the finite and the infinite, with which the larger problematic of the relationship of being and time is, as I have argued, irreducibly in communication.

In the next two sections, I will attempt to verify some of the implications of this communication of the ontological problematic with the text of mathematics in its reflective consideration of itself by drawing on the work of two philosophers who themselves consider Heidegger's problem in terms of the structurally interrelated problems of "mathematical existence": Oskar Becker and Albert Lautman. As I shall argue, the work of these philosophers verifies, in different ways, that the relevance of the text of mathematics, *including* its own internal development of the ideas of the finite and infinite, and the continuous and discontinuous, are immediately and as such relevant to the ontological problematic of being and time. This relevance is itself not new, since it is presupposed more or less explicitly in *all* "mathematical" thinking of time and becoming; but from Aristotle up to the twentieth century, it is put out of play or arrested, immobilized and held in a fixed form by the standing assumptions that Aristotle himself inaugurated: that number itself relates to time only as the counting to the counted or as the general form of calculability to what is calculated in it, that the counting of time is primarily or basically the numbering of motion on the basis of regularity whose own form is not questioned but which guarantees its calculability in general, that there can be, under these conditions, no *actual* infinite, even for thought. Along with the mathematical text itself, these assumptions *continue* to dominate thinking

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[4] But much closer, as we have seen, to their development in *Kant and the Problem of Metaphysics.*

of time and being just as much and without any *essential* change even when space is re-thought by Descartes, by means of a reconsideration of the Aristotelian physics that is also in a certain way its repetition, as pure calculable extension, and even when the physics of matter and motion is subjected to the significant internal complication of the actual inherence of the infinitesimal which appears to be a consequence of the differential calculus, in Leibniz and Newton. Nevertheless, as I shall argue, they are transformed and also displaced in a fundamental way in twentieth-century mathematical thought, first and foremost by the train of consequences that result from Cantor's demonstration of the possibility of a coherent mathematical thought of the *actual* infinite.

Many of the consequences of this demonstration are not difficult, in themselves, to observe, since they are visible in the internal paradoxes and external disputes that would mark the development of reflective mathematical thinking over the next few decades, in particular in its relation to the old question of the logical or extra-logical "foundations", which could now, in the wake of both Cantor's discovery of the transfinite and Frege's quantificational logic, be pursued under a new methodological impetus and with renewed vigor. These consequences would play out, in particular, in the structure of foundational paradoxes, such as the paradox of Burali-Forti and Russell's paradox, which were developed in quick succession from Cantor's development of set theory, in the logicist project of Frege that attempted the renewed definition of the being of number on a purely logical ground and in the form of its ultimate failure, and again in the "foundationalist" debate of the 1920s between formalism and intuitionism, which mobilizes in a direct and profound way conceptions of the mathematical being of time or the temporal being of mathematics, and was only resolved or overcome (and in a fashion unsatisfactory to both parties) by means of Godel's own profound metalogical results in the early 1930s. The relevance of the issues discussed and debated in each of these moments to the "ontological" problematic of being and time already taken up by Aristotle is direct and obvious. To begin with, and perhaps most obviously, the problem of the existence of the (discontinuous) now to the line and to continuity in general is nothing other than the problem of the continuum, which was put on a new foundation by Cantor's investigations into point sets and also led him to formulate the continuum hypothesis and labor, fruitlessly as it turned out, to prove or disprove it throughout the last decades of his life.

Subsequently, the methods developed by Dedekind and others for defining real numbers in general in terms of sets of sets led to, on the one hand, the determinacy of real numbers as such being put on a new formal basis and, on the other, foundational worries about the "impredicativity" of any such definition that were developed in a critical and limitative way by intuitionists such as Brouwer and those sympathetic with them (such as Weyl). More generally and broadly, the Cantorian demonstration of the coherence of the actual-infinite and its positive accessibility to mathematical thought sets in motion the train of formalization and reflection about mathematical proof and truth that culminates in Godel's theorems and in the idea of the nature and limits of effective processes that they bring to essential articulation. As I have already suggested (chapter 5 above) and will try to verify in the following, the ultimate implication of this development *for* the ontological problematic of being and time lies in its demonstration of the undecidable as a positive and constitutive phenomenon, and of its constitutive and problematic relationship to the logical idea of consistency, which of course determines the

determining form of specifically logical thought from Aristotle on. Relatedly but somewhat differently, the undecidability of the continuum hypothesis in terms of the foundational axioms of ZF set theory itself positively witnesses the inherence of the aporia of the continuous and the discontinuous, just where Aristotle himself attempts to block or foreclose it. Subsequently, undecidability in both senses must be taken to articulate the very structure of any structural unfolding of the finite and the infinite as such. As such it points as well to the underlying character of time, and to the actually positive thought of the undecidable as inherent in all (countable and measurable) time as such, and to its reality as an actual inherence that can no longer be treated as possibility or dunamis over against actuality as effectiveness, but is instead the structurally indicated real of ineffectivity itself.

II

The mathematically informed work of Oskar Becker and Albert Lautman on Heidegger's philosophy and the problems raised therein has received relatively little attention in Anglophone scholarship. For instance, Becker's important early work, *Mathematische Existenz*, published in 1927 in the same issue of the *Jahrbuch fur Phaenomenologische Forschung* that also contained the first publication of Heidegger's *Being and Time*, and reflecting Becker's close engagement with Heidegger arising out of his participation in Heidegger's courses and seminars in Marburg, has never been fully translated into English, and the first translation of Lautman's main works into English appeared only in 2011. Yet as I shall attempt to show in this paper, Becker's and Lautman's work on the nature of mathematical existence and the implications of mathematical practice, in constant dialogue with Heidegger's fundamental questions of the meaning and truth of Being, points the way to important new ways of understanding Heidegger himself and yields unanticipated directives and new resources for making progress with some of his most important questions. These include the question of the thinking of Being from out of, and beyond, the ontological difference, the ultimate nature of truth as unconcealment or *aletheia*, and the original structure of time as it is given both in history and in nature.

At first glance, interpreting Heidegger in close connection with the question of mathematical existence, as both Becker and Lautman do, may seem a problematic enterprise in several different ways. Heidegger himself famously and repeatedly rejected any direct application of mathematical or formal-logical methods to the problems of the meaning and truth of being, and the "Platonistic" interpretation of mathematical objects as timeless or sempiternal existences represents for Heidegger a primary and guiding instance of the tradition's interpretation of Being as presence, which Heidegger would constantly oppose. But for both Becker and Lautman, the question of mathematical existence is not primarily the question of the ontological status of mathematical entities, but rather that of the possibility of mathematics itself – that is, of the possibility of a science in which it is possible to know that which was called, from ancient times, the *mathemata,* that which can be transmitted across languages and cultures without remainder or loss, and for this knowledge to be applied to the world as comprehended by the natural sciences and understood in its fundamental regularities.

And this question in fact retains a methodological and thematic priority for Heidegger, in several different ways. First and perhaps most obviously, within the later Heidegger's interpretations of the history of Being after the mid-1930s, the contemporary epoch of the dominance of technology and enframing is seen as possible only on the basis of the dominance of an understanding of the world based on mathematical natural science and prepared through the thoroughgoing mathematization of nature. Here, the decisive possibility of a mathematically based, calculative understanding of abstract space and matter has the significance of a projective pre-understanding that makes possible the regime of the unified calculability and manipulability of beings in accordance with the overarching values of efficiency and productivity, a regime which, Heidegger suggests, represents the culmination of the metaphysical tradition in its ever-greater forgetfulness of Being. But second, and more deeply, this link between mathematization and metaphysics is by no means limited, for Heidegger, to the last historical stage of the metaphysical tradition, but is in fact already decisive in producing the very possibility of a natural-scientific understanding of the world, a possibility that is already prepared, according to Heidegger, through the conception of time and measurement that emerges at the beginning of Western thought, in Plato and Aristotle.[5]

If, in particular, already for Aristotle, "time is the number of motion," the tradition's understanding of time unfolds, throughout its itinerary, in close and determinative connection with the mathematical possibility of measurement, counting, and the givenness of a structure of numerical order, a structure that is conceived ambiguously as given from outside temporality and yet as constituted within it (for instance by the constitutive activity of the soul, the mind, or the transcendental subject in counting and measuring). From Aristotle up to Husserl, this ambiguous mathematical conception of the givenness of time determines as well, according to Heidegger, the general possibility of the epochal projection of beings by which the disclosure, sense and meaning of entities is determined in specific domains of scientific praxis and the discrete epochal regimes of the history of metaphysics; in each of these cases the fore-projection of the character of beings as determined by what appears to be a privileged entity is itself prepared by the conception of time, already present in Aristotle, that sees time as a sequence of measurable, present "nows". As we shall see, Becker and Lautman find at the basis of this Aristotelian definition a more original Platonic problematic of the givenness of time as the possibility of number and measure, the limitation of the *apeiron,* to which the obscure Platonic doctrine of the Ideal-numbers apparently answered. If this more original schematization can indeed be seen as ontologically underlying the possibility of the measure of time and the givenness of the world of nature it numbers, this points the way to an ontologically unified understanding of the basis of natural time as equiprimordial with the historical time of eventality, and in a certain way permits them to be understood (in ways that go beyond the letter of Heidegger's text) on a unified ontological basis.

In each of these ways, the problem of mathematical existence poses in a radical fashion the question of temporality itself. As we shall see, Becker and Lautman's different ways of posing this question point the way to what is in certain respects a radicalization of Heidegger's inquiry into the ontological structure of time. What most emerges from this is that the resources of a metalogical discourse whose

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[5] HCT, p. 5.

methods and reflective specificity in the twentieth century have often been difficult to place are liberated for an improved ontological understanding of the problem of projection, of the possibility of the fore-structure of understanding by which we always already have obscure access to the sense of Being in itself, and of the possibility of all ordering that always has a specific relation to the metaphysics of presence and the interpretation of Being in beings.

Oskar Becker's work *Mathematische Existenz*, published in 1927 in the same issue of the *Jahrbuch fur Phenomenologische Forschung* that contained the first edition of Heidegger's Being and Time, undertakes to investigate the "being-sense" [*Seinsinn*] of mathematical phenomena through the research methodology of "hermeneutic phenomenology," here understood as "ontology" in Heidegger's sense of a "hermeneutics of facticity" (p. 1) In particular, according to Becker, it is essential that the question of the meaning of mathematical existence be posed in relation to the structural basis of factically existing "human Dasein," which, Becker follows Heidegger in suggesting, provides the foundation for the unity of all possible interpretation of meaning. (p.1) Thus, the interpretation of mathematical existence must always refer back to the phenomenological interpretation of the mode of life in which the activity of "mathematicizing" (*mathemaitikeusthai*, analogously to philosophizing or making music) takes place and it is the structure of this life that must provide the ultimate guideline for understanding its deliverances or productions. (p. 1) However, as Becker acknowledges, since ancient times *mathema* has also had the sense of objects and objectivities existing in themselves, quite independently of human activity, and even bearing the strong temporal determination of being eternal or timeless. (p. 2) Even if this interpretation does not ultimately prove correct in the course of the hermeneutic interpretation, it has provided the conceptual context of the most usual way of interpreting the existence of an "external world" that is "obviously ruled and illuminated, in unsuspected ways, by mathematical harmony," and must accordingly be taken into account in the context of a more fundamental phenomenological inquiry into the ontological structure of the natural world. (p. 2) In fact, as Becker suggests, this inquiry may reveal the "objects" of mathematics as, in a peculiar but substantive sense, "transphenomenal," and even in a certain way "beyond being" and thus as testing the very boundaries of phenomenological interpretation itself. (p. 3)

Becker takes the basic directive for his interpretation of the sense of mathematical existence from the (then-contemporary) debate in the foundations of mathematics between the formalism of Hilbert and Bernays and the intuitionism of Brouwer and Weyl.   Because of the decisive way in which the structure of the infinite enters into foundational research through Cantor's set theory and other advances of nineteenth and early-twentieth century mathematics, the question of the nature and accessibility of the infinite is crucial to this debate and its possible resolution. For the formalist, access to formal-mathematical structures, including infinite ones, is possible only on the basis of a "proof theory" that sees mathematical proofs as, themselves, combinatorial mathematical structures that are necessarily finite; in the most extended sense, the accessibility of a mathematical structure or object (including infinite and transfinite ones) is guaranteed by the existence of a finite, non-contradictory proof within a specified, axiomatic system. This gives rise to the problem of demonstrating the noncontradictoriness of particular axiom systems and of providing axioms which allow the noncontradictory specification of infinite sets and totalities; for Hilbert, this provision allows for the Cantorian hierarchy of transfinite

numbers actually to be demonstrated and guaranteed. For the intuitionist, by contrast, no mathematical object or set is demonstrated to exist unless, and until, it is concretely provided to the actual intuition of the mathematician. Moreover, for the intuitionist, in a basic sense, "only finite discrete wholes" can be so given. (p. 7)   The infinite, for instance the infinite series of whole numbers, can be given only through what Weyl calls a "basic arithmetic intuition" of the unlimited *possible* progression of the series 1, 2, 3, 4…; more generally, according to intuitionism it is possible to give an unlimited series of natural numbers only insofar as it can be specified through a finitely intuitible formula (for instance, the series 1, 4, 9, 16… through the formula: n^2). (pp. 8-9)

Becker develops the contrast between the two approaches by considering the nature of various types of infinite series. In addition to series that we can consider to be determined by a well-defined and known mathematical rule expressible in a finite formula, we can also imagine another kind of series known as a "freely becoming choice series"["*frei werdenden Wahlfolgen*"] (p. 8). Such a series is, for instance, the series of numbers 1-6 determined in random fashion by successive throws of a die. As Becker emphasizes, it is essential to such a series that it develops only over time and it is thus nonsensical to speak of its properties except at a particular stage of its temporal development. For instance, with respect to the question whether a "6" appears at any stage in the series determined by successive die throws, it is impossible to answer this question affirmatively or negatively at any specific stage (assuming a "6" has in fact not yet appeared). This suggests to the intuitionist that the law of the excluded middle admits of "certain exceptions" in the realm of infinite series (p. 9); in particular, it is not always the case that, for a specific property E and an infinite sequence F, the statement "Either there is, in the series F, a number with the property E or there is not" is an exhaustive disjunction. Moreover, according to the intuitionist this denial of the general applicability of the law of the excluded middle can be generalized to the question of the decidability of arithmetical and other unsolved mathematical problems (for instance the problem of whether there are more than 5 primes of the form 2^n+1, or the truth-value of Fermat's last theorem, both problems unsolved at the time that Becker wrote) (pp. 11-12). According to the intuitionist, therefore, in answer to the general question of the decidability of these (in fact unsolved) mathematical questions there is nothing general to say, and it is moreover impossible to determine infinite sets and structures through a general application of the law of the excluded middle or the procedure of proof by *reductio*. This leads to what are from the alternative, formalist standpoint, severe limitations on the availability of infinite totalities; for instance, there is no sense in speaking of the totality of all number-series or indeed of most non-denumerable infinite sets, and the standard Cantorian definition of the continuum as a point-set fails within the intuitionist context. Nevertheless, Becker suggests, the intuitionist limitation makes possible a phenomenologically rich theory of the continuum and the infinite as a "medium of free becoming" ["*Mediums freien Werdens*"] (p. 13) in which, as with the freely becoming choice sets, temporality plays an ineliminable and decisive role.

Given these distinctions, Becker can now specify the positions of intuitionism and formalism on the question of mathematical existence as two specific but differing criteria for the existence of mathematical entities and structures (pp. 21-31). According to the first, formalist definition, that something is mathematically existent means that it can be made thematic within a mathematical theory

in which it can figure without contradiction; in line with this definition, a proof of the *noncontradictoriness* of an object or structure within an axiomatic theory suffices to demonstrate its positive existence. The second, intuitionist definition, by contrast, holds that mathematical existence requires *constructability* from certain basic concepts by specific means; here, therefore, the proof of the existence of a mathematical object or structure requires that its construction actually be carried out. Becker now offers (pp. 32-53) a series of phenomenologically based objections to the formalist position. First, Becker suggests, Hilbert has not motivated the link between non-contradictoriness and truth. Within Hilbert's formalist approach, in fact, individual mathematical statements are not evaluable as true or false at all, but only as consistent or inconsistent with a particular axiomatic theory. The *meta-* mathematical statement that a sentence is indeed consistent with a theory may be itself be treated as true or false, but this does nothing to support the actual truth of the sentence in an ordinary sense. Second, the analogy that Hilbert draws between the formalist's method of introducing the transfinite (by showing its non-contradictoriness) and the provision of an "intuitive substrate" for imaginary numbers by Gauss is misleading. With Hilbert's method of demonstrating non-contradictoriness, nothing like an intuitive representation of the transfinite is achieved; rather, in addition to what can be directly intuited, a second, "strange" [merkwürdige] (p. 41) kind of mathematical objectivity is invoked. Seen phenomenologically, Becker suggests, this is the puzzling category of non-contradictory objectivities that nevertheless are in principle inaccessible to what Husserl called a categorial intuition; since the availability to such an intuition is, for Husserl, constitutive of formal-ontological Being, we would be justified in treating the transfinite as understood by formalism as a mysterious third realm, in between existence and non-existence, something like what Leibniz called an "amphibium between being and non-being" (p. 41). Third, even the proofs of non-contradiction upon which Hilbert relies to permit the existence of transfinite sets rely on a strong form of *complete mathematical induction*. Becker argues (pp. 47-54) that the use of induction in this form essentially demands reasoning over a totality of proofs of transfinite as well as finite length; accordingly, the procedure essentially presupposes the actual existence of the transfinite and therefore cannot be used to demonstrate it. More generally, according to Becker (pp. 69-71) Hilbert has failed to provide a "logic of truth" capable of describing the actual *presentation* of the mathematical objectivities and states of affairs in a comprehensible givenness; instead, he has provided for the supposed transfinite objectivities, at best, a "logic of consequence" that is in fact not directed toward objects at all, but rather only toward pure "legalities," formal structures of rules, that are in themselves "impenetrable in their inner structure" and about which it makes sense to affirm neither truth nor falsehood.

Even if these objections succeed and thus constitute a negative argument for the alternative, intuitionist position, the possibility of an actual *intuitive* presentation of the infinite and transfinite is still very much in question. Becker thus turns to the question of whether and how it is possible to understand the infinite and transfinite as actually given to mathematical cognition and experience. A crucial guideline for addressing this question is given by the conceptual determinants of Cantor's own conception of the hierarchy of transfinite sets. As early as 1883, Cantor had conceived of the sets beyond the finite as forming an ordered series of actually existing infinite wholes, while at the same time categorically denying the possibility of any determination of the "absolute" or unincreasable infinity, which he identified with God. In particular, Cantor initially thought of the transfinite hierarchy as generated by

means of two "generation principles," which Becker interrogates as to their ontological significance, in close connection with the phenomenological/ontological idea of the infinite "horizon", as it had already been developed by Husserl, which makes available the "mastery of the infinite by means of a finite 'thought'" .[6] (p. 109) The first principle is that, to a present, already formed number, it is always possible to produce a new number by adding one; this is the familiar basis for counting with finite whole numbers, which Cantor extends as well beyond the domain of the finite. It is the second principle, however, which is decisive in producing the transfinite cardinals. It holds that: "If there is ever presented a succession of definite whole actual numbers, of which no largest member exists… a new number will be created, which is thought as the *limit* of those numbers, that is, is defined as the next-bigger number beyond those." (p. 109). In other words, Cantor's second generation principle allows the passage from one's grasp of the definite rule or law governing the creation of a particular unlimited series to the formation of a new number which is thought of as succeeding *all* of the numbers in the series; thus, for instance, the regularity of the sequence of natural numbers 1,2,3,4… engenders the first infinite number, ω.

In their ordered, hierarchical structure, Becker suggests, Cantor's transfinite sets still bear a certain relationship to the Aristotlian picture that sees the infinite as only existent in potentiality and constant "becoming," and thus denies the existence of *actual* infinite wholes. Seeing things this way, one might take at least the indeterminability of the "absolute" infinite, in the form of the "maximal" cardinal W, to be evidence for the fact of the impossibility of an *absolute* infinity ever to be given as an actually completed whole. (pp. 112-119). And as was shown by Burali-Forti, it is indeed impossible to suppose such a "maximal" cardinal to exist without paradox. Nevertheless, it is crucial to Cantor's conception that the transfinite sets are indeed understood as actually existent totalities, actually generated in accordance with the two "generation principles". In accordance with the second principle in particular, it must be possible to generate the limit of any well-defined series; thus the first infinite number, ω, is engendered along with the rest of the transfinite hierarchy. However, owing to the antinomy of Burali-Forti, it must be impossible to suppose the "absolute" infinite W to exist as the limit of *all* (finite as well as transfinite) counting processes.

As Becker notes, although this shows the transfinite cardinals (as opposed to the totality W) to be specifiable without contradiction, it still leaves very much unanswered the phenomenological question of their possible givenness. One option, on an intuitionist framework, would be simply to deny the possibility of any actual givenness of the (completed) infinite at all. This finitist option is the one actually taken by Weyl, who denies the actual existence of even the "first" transfinite cardinal ω (p. 88). In this "radical intuitionism," Weyl goes even farther than Brouwer, denying categorically the possibility and usefulness of anything like a general set theory. For Weyl, all mathematics is based in the actual progression of natural numbers; given a particular number, it is always possible to "create" [*erzeugen*]

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[6] "According to a basic insight of Husserl's, it is precisely in the nature of the infinite "open horizon", the "and so on", that infinitely many members "lie before us" [vorliegen] but nevertheless can be surveyed with *one* look. The mastery of the infinite by means of a finite "thought" – that is the sense of every "horizon". (p. 107)

one more in intuition, but the series as a whole never comes to an end in some completed totality. (p. 88)   Against this, according to Becker, we may assuredly consider properties of the series of natural numbers, determined by its legality, as a *whole*; thus we should sympathize with Cantor's affirmation, against finitism, of the fundamental "right" to consider the infinite. (p. 89) In particular, Cantor acknowledges that infinite totalities cannot necessarily be grasped in a single intuition; in this respect, in fact, they are like large *finite* numbers, which themselves cannot actually be *surveyed* at a single glance or within a single, intuitive act. Nor do they need, obviously, to be "actually created" by discrete finite acts from their direct predecessors in order for us to consider their properties. (pp. 89-93). Cantor thus claims that the objection that the transfinite wholes cannot appear directly "at once" in a single intuition does not bear decisively against the existence of infinite wholes. "Nevertheless we have the right to treat finite numbers, no matter how large, as objects of *discursive human* knowledge and to conduct scientific research into their constitution; *we have the same right with respect to the transfinite numbers*." (p. 89)

Thus, it must be admitted that the impossibility of bringing transfinite wholes into intuition "all at once" does not bear decisively against their existence from a phenomenological standpoint; were this objection decisive, it would bear, as well, against even the existence of large finite numbers.   In fact, as the comparison with large numbers shows, in order for the mathematical existence of the infinite wholes to be phenomenologically motivated, it is not in fact necessary that they be given "uno intentio" *as* surveyable wholes at all; all that is necessary is that there be phenomena in which they are "concretely" or "phenomenally" rather than merely "symbolically" given. But Cantor's crucial step, formulated in the second generation principle, from the reality of the (unlimited and always proceeding) counting process to that of the completed, actual transfinite sets remains, according to Becker, "extraordinary" and puzzling – is there any example of this kind of transition that can actually be drawn from concrete phenomenological research? As Becker notes, from an early stage in his work Husserl recognized the possibility that large sets whose members are not individually surveyable are nevertheless intuitable by means of a sensory "figurative moment" (*Philosophy of Arithmetic*) that allows the "running-through" of their totality in thought to be taken as completed, even without each individual being separately and distinctly considered. Later, Husserl understands this possibility of the givenness of large *finite* wholes as possible through a "categorial intuition" that yields a present but non-sensory intuition of the whole as such. In fact, within Husserl's framework even the "…and so on" of an unlimited process of counting or addition is phenomenally underwritten through the phenomenon of the "open horizon" (which, according to Husserl, has both sensory and non-sensory aspects). Nevertheless, this kind of givenness of the infinite still stops short of providing a "concrete undergirding" for the Cantorian generation principle and for the designation of the higher orders of the infinite, which, as Becker points out, one can actually pursue and carry out, at least to a certain point. It is thus necessary to find a motivation within concrete, phenomenologically understood experience for the givenness of the infinite in general, as well as for the specific givenness of at least some transfinite sets. How, then, is it possible for the Cantorian second "generation principle," which yields the successive orders of infinity, actually to be supported in concrete experience?

Becker gives two examples intended to show how it is possible, in actual, concrete experience, to pass from the awareness of an indefinitely iterable process to the actual presentation of its completion. The first (pp. 96-98) arises from the possibility of an iterated skepticism, or relativization, of specific propositions. For instance, taking a particular claim or sentence p_0, as a starting point, it is possible to doubt that sentence, or treat it as merely relative (e.g. to particular historical circumstances). We thus form the sentence "It can be doubted that p_0" or "the truth of p_0 is merely relative" We can designate this p_1 and iterate the process, up to the whole series of sentences p p_1, p_2, p_3, ...p_n, $p_{n+1....}$, etc. Grasping this possibility, we now have the series "It can be doubted, that it can be doubted, etc., etc.,...in an endless sequence" (and we may designate this, Becker says, p_ω). The iteration of the process of doubt, or of historical relativization, may seem an empty and unmotivated process, one that would in fact hardly occur (beyond, say, the second or third levels at least) in simply epistemological reflection. However, according to Becker, it can be made concrete and provided with an actual phenomenal reality if we add, at each level, the consideration that the formation of that particular level is actually a possible act of the "individual Dasein, in the way it each time [jeweilig] is" (p. 98). Thus the possibility of each level is given concrete motivation by reflexively returning, at each stage, to the concreteness of actually lived experience:

> There arises, therefore, a strong motivation to conceive arguments of this type in such a way that they gain more content. For instance, one can seek such a motivation along these lines: if one begins with the view that all concrete opinions expressed in the form of judgments *in the spiritual-history of mankind* are not "absolutely true," but rather dependent upon the specific contemporary historical situation, one can grasp these concrete judgments as "assertions of the 1st level" (A_1) and this position as an "assertion of the 2nd level" (A_2). Now, of course, this assertion A_2 also actually becomes something that also occurs in the process of spiritual history, therefore *now* and in the West, so it also belongs to the historical-concrete assertions in a wider sense. (This is no longer a purely empty game: the development of historiography, of the historical and "cultural-philosophical" consciousness, are themselves spiritual-historical processes). The consideration thereby formed is one about expressions of the second level, and so is itself of the third level. (Critique of historical consciousness). But it does not end there: one can also, in various ways, proceed to the fourth level. (For instance: Critique of the critique of historical consciousness, or history of the history of historical consciousness, and so forth). One therefore has a seemingly *contentful possibility of iteration*. The greater "contentfulness" of this last example was reached by turning back to the concretely historical, i.e. finally to one's own Dasein (*das eigene Dasein*), in the way it each time (*jeweilig*) is. If one pushes further this turning-back to one's own Dasein, one arrives at the conception of other closely related examples of the transfinite iteration of reflection in itself, related to the example of relativization. (p. 98)

A second example (pp. 98-101) is provided by the familiar kind of picture which includes an iterated reproduction of itself; for instance, the cover of a picture book for children depicts a scene in which a child is holding that very book, which in turn depicts itself again, etc. Of course, in actual pictures of this nature, only a finite number of iterations can, in fact, be included; nevertheless, as Becker points out, in imagination it is possible to conceive of the iterations proceeding into the infinite. Thus we can achieve

an imaginative or schematic (and thus not merely "symbolic" but actually *intuitive*) presentation at least of the first transfinite level (that of ω); and we can again imagine this picture to be depicted, obtaining ω+1, ω+2, etc.

These examples point to a more general possibility for the phenomenological givenness of the infinite in concrete reflection and, in particular, in consideration of the standing possibility of iteration of reflection on oneself. Husserl, in *Ideas I*, had discussed the "step-characteristic" arising from iterated reflection on experience; this is a kind of "index" that phenomenologically marks the levels of reflection, reflection on reflection, etc.[7] Though Husserl himself develops this possibility of iteration only up to indefinitely high *finite* levels, it is in fact possible, Becker argues, to derive from it an "actually living motivation" (p. 102) for a particular type of iteration of reflection which can be conceived as continuing to the transfinite level. In particular, Becker considers (pp. 102-103) Karl Löwith's development of an existentially motivated kind of reflection which Löwith finds exemplified in Dostoevsky's "Notes from the Underground" and calls the "parentheses reflection" [*Parenthesen-Reflexion*]. Dostoevsky's work presents the self-dialogue of a fallen man who considers himself and his life as he has factically lived it. This reflection is fruitless and self-defeating; at a certain point, however, "just this fact," i.e. the fact that he can and does reflect on his life (even in this unfruitful and self-defeating way) itself becomes a theme for reflection. And even *this* fact, that he is considering his own reflection, can itself become the theme of reflection, and so on. All of this, moreover, takes place within the course of the concrete, factical life that was the initial theme of the reflection; in this way, the iteration of reflection gains a concrete motivation from the structure of that life itself, and in particular from the "living motivation to flee the groundlessness and nullity of one's own Dasein and to find an inner stability by means of sincere, unsparing self-examination." (p. 103). In this way the concrete structure of this Dasein itself proves to be (in a way that, Becker suggests, can be supported by Heidegger's analyses) the actual motivation for the structurally presented possibility of the iteration of reflection to the infinite, and beyond to transfinite levels.

This description of the phenomena gains further support from Emil Lask's phenomenological description of the distinction between particular contents and the categorial forms that "encompass" them. For Lask, the relationship of form to content, which allows all determinations of "validity," is analogous to a "clothing" of material with form. This process of clothing can be iterated in reflection or in iterated validity-judgments, and it thus becomes possible that, in this iterated process, the *univocity* of the concrete steps of iteration is recognized. (pp. 104-106) Analogously, in the case of the parentheses reflection, one can in fact recognize that the impulse to take "flight" before facticity has no end: in this recognition, the impulse to flight itself drives "out over the infinite" [*über das Unendliche hinaus*]. But this consciousness of the possibility of unlimited, univocal iteration through *all* finite steps is itself, according to Becker, the ω*th* step, and it is now possible to continue to the ω+1st, etc. In this way the givenness of the infinite receives structural motivation from the concrete possibilities of factically

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[7] p. 101; Becker references, in particular, sections 38, 77, 78, 100, 101, 107, and 112 of *Ideas* I.

experienced life, and "one actually finds…a way from concrete, 'historical' life-motivations to a *transfinite* iteration of reflection."[8] (p. 106)

The possibility of this kind of awareness is in fact already implicit, Becker suggests, in the phenomenon of "horizon" described by Husserl; for it is precisely the nature of the "open horizon" that it yields a structure in which an *infinite* number of elements can be, in a certain sense, potentially present or ready-to-hand but which can itself be surveyable in a *single* "glance". In this sense, the meaning of the epistemological "horizon" is nothing other than "the mastery of the infinite by means of a finite 'thought.'" But with Lowith's parentheses-reflection, Lask's iteration of validity judgments, and the general possibility of grasping the standing possibility of iterating reflection on one's concrete life-situation, one gains, according to Becker, a structural motivation for the actual availability of the transfinite that is not merely "epistemological" but actually concretely rooted in the structure of factical life itself. In particular, it is in consideration of the tendency to flee one's factical existence, revealed by, these standing possibilities of concrete reflection, that the structure of the transfinite first appears and becomes concretely motivated as a structural *feature* of that existence itself.

Although the law that determines the transfinite process of reflection in its iterability up to the transfinite it itself, "according to its content," finite (as, Becker says, is every content of consciousness), it provides the reflective basis on which it is first possible phenomenologically and formally to demonstrate and grasp the *whole* of the infinite series that it permits.

The formalization that indicates this structural inclusion of the infinite in the form of Dasein's factical life is, according to Becker, to be sharply distinguished from any kind of abstraction. In fact, the method of this demonstration is, Becker suggests, nothing other than the Heideggerian device of *formal indication*, whereby what is concretely given in factical life is hermeneutically investigated according to the underlying ontological ground of its givenness in the *singular* [jeweilig] case.[9] In the phenomenological

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[8] Becker here considers a possible objection: as a matter of what factually occurs, we continue the process only up to a *finite* step, say n, and then *realize* that the steps can be continued indefinitely. This realization (so the objection goes) is then only the *n+1*[st] act; thus only n+1 (a finite number of) acts have actually taken place. However, (Becker answers), we need to distinguish the *contemplation* of the iterated inscription from the iterated inscription itself. The former is indeed finite, but the latter is *structurally* infinite. It is in this way that the finite and factical process of reflection indicates, in itself and *as* a process, the specifically infinite structure.

[9] "But it seems to us that the sense-analysis (hermeneutic analysis) of facticity can only do justice to facticity itself if it treats it in *its individuality* [*ihrer Jeweiligkeit*]. This individually existing [*jeweils so und so seiende*] life has such and such characteristic features which can and must be presented *in a certain generality*. But this requires a special kind of conceptualization, namely a "formally-indicative" (Heidegger) kind, whose "generality" lies in its directedness to the "in each case" [*Jeweilige*]. One can *also* call this "formally-indicated" being-Sense "essence" – but this "essence" is then basically different from any "*eidos*" (especially from any material *eidos*!)[9]

For us, this requires, therefore, an *interpretation of factical Being* [*Auslegung faktischen Seins*.]" (p. 183) Becker also says here, in a footnote, that one can speak of this as a disclosure of the "transcendental", provided that "transcendental" is used "more in a medieval than a Kantian sense," and identifies this sense with that in which Heidegger says in *Being and Time* (section 38) that "Being is the *transcendens*, pure and simple."

demonstration, the ultimate basis of the possibility of the formation of Cantor's transfinite sets, and at least some aspects of their ordering, becomes evident in "constructivist" terms (in accordance with Cantor's two generation principles) through reflection on the concretely experienced process of iteration of reflection. However, *by contrast* with the usual process of mathematical formalization, which results in a certain categorial "leveling" whereby, for instance, a dynamic sequence of iteration is "flattened out" into the static character of a set, here the grounding of the demonstrative method in concrete, hermeneutic experience allows the essential structure of iteration to be preserved, along with its dynamic, temporal character of succession. In the formal indication of the iterated series, in particular, the "sign-function" of indication is not a generalizing abstraction, but is *itself* a "particular comportment," i.e. an actual directedness to the matters at hand[10]. Thus, for the structure to be indicated, it is thus not a question of finding a general symbolization, or a static mathematical abstraction of the usual kind; rather the "sign-function" here remains of a "dynamic" type and thereby retains the sequential and temporal structural aspects of what it indicates. [11] (p. 192)

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[10] Cf. pp. 191-192: " Relative to authentic substantive claims, in life and in descriptive science, every mathematical grasping of a thing, which is closely bound up with mathematical designation (e.g. by letters), is a bodily-holding-oneself away from the object, a turning-oneself away from its actual Being. The thing becomes, in a certain sense, a bare holding-point, toward which the thread of a (generally peculiarly empty) intentionality is directed. All that is actually in view is the "object of a higher order," which is structured on the basis of the (in itself now indifferent, therefore "leveled") intentionality of the first order. These intentionalities of the first order form possibilities, by way of their equivalence, for a multiplicity of relations to be ordered with respect to one another ("multiplicity of references" (Heidegger)). The possibility of "flattening" the references in this sort of multiplicity distinguishes the mathematical-formal, in an ontological respect, from the formally-indicative. For in the "formal indication" (Heidegger) the reference-sense remains in suspense; it is not fulfilled, but also not displaced (i.e. "abstracted" (through *aphairesis*), but rather constantly is in readiness, at the ready, in a certain sense, for the *search* for fulfillment. The sign-function, the indexing in formal indication is a particular comportment; it is of a dynamic type. By contrast, the mathematical form is in itself quiet, autarkic, static; indeed, capable of fulfillment, but in no sense requiring it."

[11] "The mathematical-formal of the traditional type is recognizable precisely through this leveling of categorial differences, as opposed to the "formally indicated" (Heidegger) which cleaves to the peculiarity of the phenomenon-body like a thin plastic garment. On the other hand, even this formalization of the "indicative type" is capable under certain conditions of a certain kind of mathematical treatment. This is perhaps surprising, when one considers that the formalization of the indicative type is actually the methodological ("conceptual") basis of the interpretable human sciences [*Geisteswissenschaften*] (philology, history, and so on). In any case it is to be emphasized that this possibility of mathematically explicating the formally-indicated characters does not hold in general, but only has its ground in any specific case in the *iterative* structure of the phenomenon. Here, the possibility opens up of a deep insight into the essential character of mathematics in general, in which the iteration, "repetition" of whatever is in any respect the *same* (or *identical*) plays a decisive role." (pp. 125-126).

In the formal indication of the transfinite levels through consideration of the structural possibility of iterated reflection, the series of these levels is itself thus indicated through a *particular concrete comportment* that preserves the dynamic and temporal character of the progression as it is actually given in concrete experience. In this, according to Becker, one can see again the reflexive (or hermeneutic-circular) structure of hermeneutic reflection on factical life, which is again *itself* an activity within factical life. What is thus formally indicated is nothing other than the underlying ontological structure which gives rise to the possibility of reflexive iteration itself, as this is implicated in, among other things, the iteration of symbols in mathematics; indeed, according to Becker, in the formal indication of the iterative series as such, "the possibility opens up of a deep insight into the essential character of mathematics in general, in which the iteration, "repetition" of whatever is in any respect the *same* (or *identical*) plays a decisive role." (p. 192).

In this way, concrete, phenomenological research, underwriting Cantor's two generation principles on the basis of actually lived facticity, provides a phenomenological demonstration of the actual existence of the series of transfinite cardinals, at least insofar as they are "constructible" (in accordance with Cantor's second generation principle) by means of a successive grasping of the law underlying each successive, unlimited series. What, though, of the *totality* of the transfinite hierarchy (what is today sometimes called the "set-theoretical universe," V), or the "maximal" ordinal W? Can we consider this totality itself to be given through the Cantorian principles as underwritten by the concrete, hermeneutical analysis? In fact, Becker argues, we cannot. For the leap from a lower to a higher transfinite level is accomplished in each case only by reflection on the legality and regularity of the series conceived as coming "before"; in each case, it is only by considering each member of this series as generated by a law that is, in a certain way, "always the same" that we attain the essential insight that allows us to move up in reflection to the next, higher level. It is, thus, only in concrete reflection on the law determining a particular series that we are able to move up the ladder of transfinite cardinals. In the case of the (paradoxical) ordinal W, however, there is *no particular series-law* which can support such a reflection. In fact, in order to form the ordinal W in accordance with the phenomenological demonstration, it would be necessary to reflect concretely on the totality of all *possible* series-laws; but this is not possible, according to Becker, since there is no such totality. In fact, in the ordered series of transfinite cardinals, each successive series-law builds on earlier ones, but the process is "in no sense ever given in its completeness;" rather, "it is always grasped in becoming (*dunamei on*)." (p. 112) The hermeneutic demonstration, despite showing how the actual infinities of the transfinite hierarchy can in fact be generated on a factical basis, thus preserves with respect to the *absolute* infinite the intuitionist's basic conception of an unlimited and free becoming. This verifies Burali-Forti's insight into the paradoxical (and thus impossible) character of the totality W, while also vindicating the Cantorian conception of the accessibility of the transfinite cardinals in a process that nevertheless never reaches the absolute.

In fact, the circumstance that the totality W itself is not given, and cannot be given, as the limit of any definite series-law itself points, according to Becker, to an inherent complication in the transfinite process of reflection itself, which in turn provides the occasion for the re-introduction of a certain element of "freedom" and temporal futurity into the concrete process of reflection and the dynamic

structure that supports it. In particular, as the contemporary work of Mahlo and Veblens had shown, in the development of the transfinite sequence it is necessary again and again recursively to define new symbols and new principles of connection between them; in so doing we must recurrently, at new transfinite stages, introduce "certain new forms of manifolds." Expressed in the terminology of intuitionism, this means precisely that the "*sequence of successively presented formal series-laws" is itself a "becoming sequence, whose "future" is not forseeable in advance"* [*eine werdende Folge, deren 'Zukunft' nicht voraussehbar ist*]. (p. 112). At each stage of successive complication, in other words, reflection on the legality of the series at a previous level requires what is genuinely a new and creative formation, one that is not mechanically determined at the level below. This is why the totality W can never be thought as the outcome of reflection on the limit of a particular law; instead, according to Becker, the "*number W corresponds only to the wholly indeterminate "free" horizon of the transfinite process continued on endlessly through the two generation principles.*" (pp. 112-113).

Because of this, it is possible to see in the transfinite development of the hierarchy of reflection, as determined by the two generation principles, an essential element of freedom that goes essentially beyond what was achieved by the initial "breakthrough" to the transfinite level. In this initial "breakthrough," what guaranteed the progress of insight "out over infinity" was the insight that the stages of iterated reflection are all, in a certain sense "the same"; this amounts to the initial grasping of the series-law that determines them. Here, the infinite totality thus simply has the sense of the totality of a series determined as the law-bound repetition of the same; Becker compares this to the "bad infinity" of Hegel (p. 113). But in the further development of the transfinite hierarchy, it is essential that the formation of new stages, each of which includes but in a certain sense goes beyond what has come before, involves new creative activities and cannot be thought of as completely determined by the regular process of the foregoing. Accordingly, Becker suggests, it is possible to see in the whole unlimited progression of the transfinite, which (as is shown by the actual unavailability of W) can never be completely schematized by a rule, the essential structure of a temporal process allowing a constantly unfolding future that cannot be predicted in advance:

> One guesses that the philosophical (that is, ontological) final meaning of the transfinite process thereby brought to givenness would consist in the fact that in it the phenomenon of the endless, of the foray into the unknown future, comes to its highest conceptual formation – that in it the formally-indicated schema of the true infinity comes to appear, [a schema] which never may – as Hegel so often forcefully expressed – be confused with the familiar "bad infinity" schematized through number series. (p. 113)

In this complication of the transfinite series, which thus requires at each stage new creative acts of innovation, Becker thus sees the possibility of a motivated structure of transfinite becoming that does not amount simply to the lawbound infinite repetition of the same, but actually brings into existence a genuinely creative and unanticipated future as underwritten by the possibility of concrete reflection given in the actual structure of factical life itself. The necessity for new creative acts at each stage demonstrates the genuinely open character of this ongoing temporal process of reflexive becoming, while its actual uncompleteability, in accordance with what is abstractly demonstrated by the Burali-Forti antinomy shows that the process of hierarchical reflection can never come to an end. According to Becker, this witnesses the ultimate impossibility of life "actually fleeing before itself" and thus

corresponds to the "radical groundlessness" of the concrete life thereby reflected on (p. 113). This groundlessness means, in turn, that in concrete life the individual Dasein is always free to determine itself in the unfolding of a temporal process that can never be completely captured by rule.

Returning, then, to the contemporary debate between intuitionism and formalism about the possibility of the givenness of the infinite, Becker concludes that the phenomenological-hermeneutic analysis decides this debate unequivocally in favor of intuitionism. In particular, it provides a description of the actual givenness of the transfinite hierarchy, in unlimited becoming, that does not deny the reality of the vast reaches of Cantor's hierarchy but nevertheless motivates their existence concretely as an aspect of the structure of concrete Dasein itself. By contrast, Hilbert's formalism, which sees the existence of mathematical objects as guaranteed simply by their freedom from contradiction provides no such motivation, and is accordingly to be rejected. To show this more specifically, Becker considers the different ways in which the functioning of signs in mathematics is understood by formalism and intuitionism. Hilbert's formalist mathematics is often discussed (perhaps somewhat misleadingly) as treating mathematics as a combinatorial game played with "signs that mean nothing;" in fact, Becker suggests, the phrase "sign that means nothing" is a contradiction in terms, since a sign without meaning (in the sense of being "denuded of [its] indicative function") is not a sign at all. (p. 193) But there is nevertheless something right about the phrase, in that it captures the way, in mathematical demonstration, the referential meaning of signs is held "at arm's length" and demonstration proceeds, first and foremost, through the manipulation of the signs themselves. Thus, "the ontic accent falls on the reference" [*Bezug*] and the possibilities for the manipulation and intercombination of signs in a certain sense dominate and even determine the properties of their referents. (p. 194) accordingly, here, the noesis is in a certain way "primary" over the noema (and this is quite different from, for instance, the structure of perception, in which the opposite is the case). In mathematics, as distinct from other cases of signification, the activity of synthesizing signs (for instance in an "existence proof") thus in a certain sense "produces" mathematical objects, rather than their simply being "encountered" as ideal objects in the world of ideas. This is the legitimate core of insight in Hilbert's formalist approach, according to which the regular, noncontradictory synthesis of signs *alone* is sufficient to guarantee existence; to this, of course, the intuitionist adds the notorious requirement of actual intuitability or presentability in concrete intuition. But what this requirement actually comes to, according to Becker, is that already-meaningful signs can *in fact* be synthesized in such a way as to refer to an "objectivity of a higher type." (p. 194)

Thus, although the referential meaning of the actual signs can never be ignored, as it is in Hilbert's approach, the weight of the demonstration of existence, and hence the *provision* of an actual reference, is borne by the demonstration of the actual executability of certain referentially meaningful "syntaxes" of signs, the demonstration that certain kinds of combinations of meaningful signs to produce combinations with the referential meaning of a "higher" objectivity indeed can be carried out. In other words, in mathematics, "the source of the ontic "vis" (the Being-power) of mathematical phenomena lies in the execution of mathematical syntheses (syntaxes)," and this provides the key to the resolution of the formalist-intuitionist debate in favor of the intuitionist. For:

…if the ontic "vis" of the mathematical lies in the execution of syntaxes, then these syntaxes must be in a stringent sense *factical*, that is, *they must be able to be actually executed*. But the "transfinite" syntaxes obviously *cannot* be executed. Hilbert's transfinite axioms express the requirement of *de facto* unexecutable syntheses …

With this, the phenomenological analysis as hermeneutic, i.e. as interpreted from out of the Dasein, decides the disputed question about the definition of mathematical existence in favor of intuitionism. For the intuitionist requirement that every mathematically existing object must be able to be "presented" through a construction completeable *de facto* and *in concreto* contains nothing other than the postulate: all mathematical objects shall be able to be reached through factically completeable syntheses. And that says, more accurately expressed: genuine ("existing") mathematical phenomena "are" only in factically completeable syntaxes. (p. 196)

In Hilbert's formalist approach, the mathematical existence of the transfinite sets is seen as guaranteed simply by a proof of the noncontradictoriness of certain symbolic formations within a particular axiom system. But such a proof does not guarantee that the actual phenomenologically founded synthesis that would be needed to motivate their existence *can* actually be carried out; and in fact in many cases, owing to the necessity of transfinite induction to carry out the proof, it is clear that it cannot be. In particular, in these cases, it is impossible to consider the relevant infinite wholes to be "actually formed" on the basis of syntheses that can actually be *de facto* carried out. By contrast, within the intuitionist framework, the demonstration of actual mathematical existence always amounts to the demonstration of the factical completeability or executability of a combination of referentially meaningful signs; this demonstration allows the combination to sustain its own, distinct reference and is in fact, Becker suggests, the whole significance of the intuitionist requirement of the actual "presentability" of the objects in question. And the actual, factical completeability of reflective syntheses that *de facto* allows the progress of reflection into the infinite underwrites, as we have seen, its actual existence and its continuation into the transfinite.

More broadly, Becker suggests that the requirement of factical executibility provides grounds for resolving what he sees as a larger dispute, within which the local dispute between intuitionism and formalism is situated, between what he calls an "anthropological" and an "absolute" conception of knowledge overall. (p. 185) For the "anthropological" conception, "man (or, better: the factical human Dasein) stands at the heart of the philosophical problematic" and the phenomenon of "world" is seen as "interpretable proceeding from the sense of [man's] factical Dasein, his facticity, from which all other facticity in the world first gains its meaning [*ihre Bedeutung gewinnt*]." (p. 185) By contrast, for the "absolute" conception, "the world is there as the universe of Being "in itself", supplied with a particular organizing structure, following certain formally general laws "according to essence"." (p. 185) Becker suggests that this distinction between conceptions underlies, in a certain way, the familiar debate between idealism and realism, but in a certain sense also goes further. For whereas the familiar forms of idealism, including Husserl's transcendental idealism, interpret intentionality only in the sense of an idealized "pure I" or "I-pole" that does not bear an essential reference to concrete life (p. 186), the "anthropological" position as Becker describes it (following Heidegger) conceives of the historical Dasein as essentially "historical" and of its basic comportment as "care"; here "the concept of the pure I or pure

consciousness disappears in favor of the concrete, full, authentically ontological questioning." (p. 187). This concreteness of the "anthropological" position allows the "constitution problem" of (Husserlian) eidetic-transcendental phenomenology to be replaced with a problematic in which "the objects of the world now appear as objects of care (in different ways: e.g. as already present material, as workable, as tasks, as obstacles, etc., etc.)" and "the "how" of their meaningfulness for life is what the actual manner of the "constitution" of the object achieves." (p. 187)

From this perspective, it is thus possible to reconsider the two definitions of mathematical existence given by the formalist and the intuitionist, respectively, and in particular to consider the particular mode of "care" which each implies. As we have seen, the formalist considers mathematical existence to be guaranteed simply by freedom from contradiction within an axiomatic system and by the unlimited possibility of continued deduction that this allows. Thus:

> If one brings into view the meaning of the demand for freedom from contradiction and asks what kind of care, or more specifically what mode of care, stands behind this demand, we receive immediately the answer: care about the unlimited continuation of deduction itself. That means, also: care about the preservation of the specific mode of care which is alive in the formal-mathematical researcher. And insofar as one holds to this preservation, one ignores the question of actual existence: that something is free of contradictions says nothing about its being-sense; it says, by contrast, precisely: one can ignore this being-sense; the question about this being-sense is wholly irrelevant for the progress of mathematical science. Thus, care here does not ask about mathematical objectivity in the how of its being; it is not concerned with Being, but only with its own being-maintained….In fact, here the "idea" of mathematical existence, paradoxically, includes the possibility of a defense against being able to ask about the "existing" objectivity in its Being. (pp. 188-189).

By contrast with this, the intuitionist standpoint requires in each case, more than the simple possibility of continued deduction without contradiction; at every stage of demonstration, the actual existence of the mathematical object must be guaranteed by an actual carrying-out of the relevant synthesis, and thus the "being-sense" (i.e. the meaning of the being) of the mathematical object remains in view and is always again founded in factual life. Thus, the requirement of factical executability (the requirement that "the intuitibility of mathematical objects rests finally on the factical executability of the corresponding syntactical noeses") decides in favor of the "anthropological" rather than the "absolute" conception of knowledge:

> With this, a certain position is finally won with respect to the alternative posed earlier between the "anthropological" and the "absolute" conceptions of knowledge. Through the fact that from the uniqueness of mathematical phenomenon the necessity arises of placing execution at the center of the analysis, the individual [*eigene*] (historical) human Dasein is decisively included. Mathematics thereby is revealed as having an "anthropological" foundation. Not a measuredly structured *universum*, "objective" in the traditional sense and existing "in itself" (as even the newest metaphysics, in whatever form, takes it to be), but rather the factical life of humans, the in-each-case-one's-own [*jeweils eigene*] life of the individual (or at least the occurrent [*jeweiligen*] "generation") is the ontic foundation, thus also the foundation for mathematics.

In this way, the phenomenological inquiry into mathematical existence itself motivates, at least with respect to mathematical objectivities themselves, a thoroughgoing foundation in Dasein, linked to the requirement of actual presentability, i.e. executability of the relevant syntaxes. However, Becker sees this result also as having profound implications for the consideration of the interlinked issues of time, decision, and finitude themselves. In particular, it is now possible to consider the implications for Hilbert's "decision question", the question of the existence of problems that are not capable of solution by finite means.

> This expresses itself in an especially heightened form in the "decision problem", which stands, not accidentally, at the center of the mathematical logic of intuitionism. For this problem is specifically human or at least a problem for a "finite" nature (a "creature"). For Kant's "intuitive intellect" (intellectus archetypus) it would not exist. God does not need to count.   (Contrary to Gauss's opinion). *Counting is much more conditioned by the essential time-boundedness of humans (more exactly their "historical" confinement)*, just as Kant had indeed already referred number back to time, that is, to what is according to him a specific human form of intuition. (section 6c, III D.) That something like a choice sequence arises step by step in time and cannot be surveyed in one moment in its whole endless extent is a direct consequence of our time-boundedness.
>
> Thus arises the task of investigating the standing of mathematical objects with respect to *temporality*, this exquisitely *human* moment of Dasein. (p. 197)

After developing the problem of the ontological meaning of the transfinite progression, Becker next takes up the closely related problem of the constitution of the *continuum*. Although this problem is, as Becker suggests, in fact one of the most ancient problems of mathematical-philosophical thought, the phenomenological demonstration of a concrete, factical underpinning for the givenness of at least part of the Cantorian transfinite hierarchy raises the question in a new light. Becker develops the history of the problem, leading up to Cantor's identification of the continuum with a point set and his proof, through diagonalization, of the excess of its cardinality over that of any countable set. However, according to Becker, this development can only be seen in its proper ontological significance by returning to the way in which the problem of the nature of the continuum, as it appeared particularly in the problem of the nature and definition of irrational magnitudes, already played a crucial role in the methodological and thematic reflections of Greek mathematics and philosophy.

According to Becker, for the Greeks in general, the method of construction had an unquestioned priority for the demonstration of mathematical existence; the demonstration of the actual constructabilty of irrational lengths, such as that of the diagonal of the square, posed challenges to this method but did not prevent the detailed classification of these lengths by mathematicians such as (the historical) Theaetetus and Euclid:

> For [Euclid], the classification and construction of irrationals was so important because it showed how one could, by demonstrating the existence of these "quantities", progress step-by-step away from relations expressible by rational numbers. The unheard-of surprise that the discovery of the irrationals made is well-known – even if the story that Hippasos, as the betrayer

of this secret of the Pythagoreans, was killed by divine force in the sea is a later invention. That something should "exist" which is not expressible by means of relations to whole numbers, something which, when one attempted to express it through the whole of number-relations, showed itself as unending (*apeiron* unlimited), seemed to threaten the thoroughgoing rule of form, the principle of order, and indeed not only in the realm of sensory, fluctuating becoming, but even in that of precisely construable Beings (those that can be grasped by *dianoia).* The difficulty nevertheless was overcome through the precise construction and classification of the irrationals themselves; a new domain was ripped from out of Chaos and incorporated into the Kosmos (cf. Plato, Philebus 16c-18c). (p. 136)

This classification of the irrationals as lengths or magnitudes, in accordance with the constructive method, thus brought a certain measure of reassuring order back in the face of the threat posed by the apparent existence of numbers that cannot be expressed as ratios. Additionally, Becker suggests, the Greeks avoided conceiving of the continuum explicitly as anything like an actually completed whole or set of individual points or magnitudes, and thus stopped short of applying to the problem of the continuum anything like set theory in the modern sense (p. 144). The most central reason for this avoidance, according to Becker, was the desire to avoid the paradoxes and contradictions about motion and becoming that had apparently been demonstrated by Zeno to be inevitable, given the assumption that a continuous span is constituted out of an actually existing infinity of points. (p. 144) The reaction to Zeno's paradoxes of the actual-infinite developed, in antiquity, in two directions (pp. 144-45). First, the atomists developed a finitist doctrine of the continuum as divided into a finite number of very small but themselves indivisible line-segments or line-atoms; and second, drawing on precedents from Eudoxos and others, Aristotle developed, in accordance with his own doctrine of the non-existence of actual infinities, a conception of the continuum as merely potential and as coming into existence only through the successive progress of actual division of the line into smaller portions. Thus, for Aristotle the continuum is not understood as an actual, completed infinity, but rather is always understood as in a process of becoming.

This Aristotelian treatment of the continuum is closely connected, on the one hand, with Aristotle's understanding of the *apeiron* always only potential (and the correlative denial of actual infinities) and thus as given only in processes of division and counting; and on the other with his specific conception of the relationship of number and time. (pp. 145-47) In particular, according to Becker, Aristotle's accommodation of apeiron as potential arises in the context of a historical tradition from an anciently rooted and mystical conception of number as figure to the conception decisive in Aristotle's thinking and indeed in all subsequent mathematical investigations into number and the continuum, that of number as seriality and order (p. 200). On the initial, "mystical" conception, still present in Plato, number is a kind of figure that gives the possibility of measure; Becker suggests that this shift is at the root of Aristotle's polemics against Plato's idea-numbers (p. 201). Here, infinite number is basically unthinkable; for the figural character of numbers is basically understood as its being limited.(pp. 201-202) On the other hand, as Aristotle certainly grasped, the conception of number as a position in a series immediately demands the thought of the possibility of an endless procession, one that can be

continued indefinitely without running to an end. Aristotle in fact sees in this endlessness a basic link not only to number and the mathematical, but to the "basic phenomenon of time." (p. 202)

As we have seen, above, time is for Aristotle i) basically continuous and becoming; ii) capable of measurement. It is through the definition of time as measurable in the sense of counting, Becker suggest, that Aristotle "tames" the *apeiron* and makes it tractable (understands its thinkability) in terms of the repetition of the same. But there is in fact a basis for i) which goes much deeper, although Aristotle does not thematize it and it appears in Aristotle's text mostly through his polemical criticism of the late Plato. The thought of time as the continuity of the *apeiron,* prior to and before the possibility of measurement has a deeper provenance in the linked conception of time and number already appears in the somewhat obscure Platonic conception of the "unlimited Dyad" as at the root of both the existence of "number" and its "generation" in accordance with a temporal or quasi-temporal anteriority, ordering of before and after.

> The most decisive examples of the unlimited, i.e. the concrete phenomena in which we encounter it, are number and time. With this we can also include the analysis of magnitudes into parts, which belongs as an analogous phenomenon to number (*synthesis* parallel to *diaresis*).

> In this sense, the late Plato had already considered something like doubling and halving in a parallel way. Whole numbers as well as fractions originate through the genetic possibility of the *aoristos duas* (the unlimited dyad). Stenzel has made it appear very probable that the diaretic development of number generates the whole of the natural numbers as well as the (binary) fractions according to the same principle.

> This opinion of Stenzel's is anticipated by Alexander Aphrodisiensis as well as by the commentary of Simplicius on the passages of the Physics about the apeiron (III, 4, 203a15). (citations): the most remarkable aspect about this discussion is that it shows that essential determinations of the Aristotelian conception of the infinite already pertain to the "unlimited dyad", such as the *ateleutetos* (uncompleteable, endless) *proienai*, *prochorein eis to tes apeirias aoriston*, the *adialeipton*, etc.

> One can thereby conclude that already in the late Plato the progression from number as figure to number as series-member is developed, in close connection with the concept of the *apeiron* as something becoming [*eines Werdenden*]. The unlimited dyad is, according to him, the potency *generative* [*erzeugende*] of number (*duopoios*).

There is thus reason to take the Platonic synthetic-diaretic generation of the numbers from the unmlimited dyad as already answering to the basic problem which Aristotle also formulates: that of thinking the possibility of a ordering, of the "before and after" of time.

If the connection is legitimate, then the character of time as it is developed by Aristotle has distal roots in the late-Platonic conception of the *ideal genesis* of numbers, according to which, as we know from Aristotle's reports, ideas are themselves in some sense to be identified with numbers and both are explained as resulting from a process of generation or coming-into-being in relation to the more basic principles of the *aoristos duas*, or the "great and the small," and unity. In the later (1931) article "The Diairetic Generation of Platonic Ideal Numbers," Becker, developing suggestions made initially by Stenzel, argues that the generation of numbers can be considered actually identical to that of the diairetic definition of a concept by division. Stenzel had suggested, in particular, that the positive whole numbers may be seen as generated by means of a process of successive binary "division," whereby each number *n*, beginning with 1, generates *2n* and *2n+1*. According to Becker, although this solution tends in the right direction, it does not explain how "ideal" (as opposed to familiar mathematical) numbers can actually *be* ideas, and it also does not explain how ideas can thereby be thought as *dynamically* generated rather than simply recovered subsequently by analysis. As an alternative, he suggests that the ideal numbers are generated by a repeated process whereby one divides into two, but in the division the original one is "sublated" or overcome in the division. In this way, the powers of 2 (2, 4, 8, 16, etc.) can be thought of as generated by the symmetrical iteration of binary division itself, while all other numbers are seen as arising from an asymmetrical development of a diairetic tree structure (e.g. 3 is generated by the division of an initial unit, a, into two, (b and c) and the subsequent division of c into d and e, while b remains unaffected; the remaining (unsublated) elements are then three (b, d, and e).) In this way, the actual seriality of number can be seen as generated in a way that is "formally identical" to the structure of the diairesis of concepts that Plato suggests in the *Sophist* and the *Statesman*; Becker further suggests a structural analogy to the Hegelian logic of "sublation" (*Aufhebung*) in the actual temporal generation of idealities and ideal relations. According to the solution, what corresponds to the diairetically disclosed "parts" of a defined concept in the ideal number is not the number itself but its "units" (e.g. b, d, and e in the example above); in this way the formal structure of the decomposition of the idea in the definition is exactly formally analogous to the generation of the number. Becker also notes the possibility of connecting this to the structure of the division of a continuous quantity by iterated fractional decomposition to produce an exact (rational) point; in this way, the process of diairesis which results in the identification of the constituents of an idea as "monads" or "ones" may be thought to produce examples of the sort that Plato appeals to in the *Philebus*, e.g. the identification of the fixed letters or discrete musical notes from the fluid continuum of possible sounds.

We shall return to this suggestion of a connection in Plato between the genesis of numbers and ideas, as this connection figures in Plato's consideration of the problems of becoming, in section V, below. For now, though, before leaving Becker's *Mathematische Existenz*, it is useful to note one further suggestion that Becker makes there: that the conception of the infinite as potential which yields Aristotle's understanding of time as the number of motion may be seen as having even deeper roots, before Plato, in the thinking of the Pythagorean Archytas, as well as the pre-socratics Anaxagoras, Zeno, and finally Anaximander. In each of these thinkers, according to Becker, the question of time already played "a decisive role in the definition of the apeiron." For Archytas and, before him, Anaxagoras, the existence of space and entities already had, in itself, the character of the aei, or "always", of eternity. We can, according to Becker, apparently trace to Zeno the first clear understanding of this *aei* as implying the

infinite repeatability, in principle, of the individual instance, as well as the idea of the infinite divisibility of continua of motion and space which yields his notorious paradoxes of motion. And before all of these, of course, Anaximander understood the principle (or *arche*) of things as the *apeiron*, or the unlimited, holding (in what may be the first direct quotation that reaches us from the pre-Socratics) that:

> "Where their arising is from, therein arises also their strife, according to necessity. For they count against one another strife and compensation according to the ordinance of time."

According to Becker, we can already see in this the origin of the conception of time which dominates Greek thought, a conception of time as the eternal, rhythmic alternation of birth and death and the repetition of the same:

> The eternal alteration, which necessarily must be thought as rhythmic, the eternal wave-crashing of birth and death – this is the *apeiron* as principle [*Prinzip*] (*arche*). [Anaximander's] statement is thus not to be thought moralistically; rather it posits a search for the expression of the endless world-process – with words that are taken from living-significance. The *apeiron* is the original power (compare Diogenes von Apollonia) that becoming never allows to cease.

In this respect, according to Becker, the Anaximander fragment already yields the prototype for the interlinked conception of the infinite and time that comes to the fore in Aristotle's developed conception of time as the number of motion. In particular, for Anaximander as well as the other pre-Aristotelian thinkers (including Plato), the sense of infinity originates from the aei, or the "always," specifically understood in each case as the eternal repetition or recurrence of the same. This endlessness of repetition is also already thought, in these original conceptions, as the decisive feature of number; and hence number is brought into a "completely original connection with time" and understood, as part of an unlimited series, as having within this unlimited horizon the character of temporality. More specifically, however, according to Becker, in all of the Greek conceptions, the relationship of number with time is thought as the eternal *repetition* of the same, the "moment" or "now" infinitely repeated in its presence. However, the previously undertaken analysis of the transfinite structure of concrete, factical reflection on facticity provides, according to Becker, a radical alternative to this. In particular, according to Becker results of Cantor, Mehlo and Brouwer demonstrate in the development of the transfinite hierarchy, at each stage, an essential *openness* that cannot be understood on the basis of the eternal repetition of the same. Here, the "progress" into the future is no longer understandable on the basis of an eternally existing substrate of present moments, each one in principle the same as the last, but as an irreducibly dynamic process of open, reflexive becoming, which Becker designates as "historical temporality." This temporality is further, according to Becker, to be seen as connected or identical to the "authentic" or primordial time that had already been

described by Heidegger as the reflexive structure of Dasein through which Dasein "gives itself its time" and is in a certain way "time itself."[12]

III

As we have seen, Becker's investigation of mathematical existence, conducted under the twofold condition of the history of Greek mathematics and the then-contemporary "crisis" of the foundations of mathematics, points to an original connection between the givenness of number and time, given and further specifiable on the level of the "ontological" problematic of Dasein. Another thinker who develops the suggestion of such an original connection of number, time, and mathematics to Heidegger's ontological problematic is the French philosopher-mathematician Albert Lautman. Despite exerting decisive influences on the work of recent and contemporary French philosophers such as Deleuze and Badiou, Albert Lautman's penetrating considerations of the nature of mathematics and its relationship to the world have, so far, received little attention in the literature on Heidegger or, indeed, in their own right. But like Becker's work, Lautman's investigations provides essential directives for an understanding of the basic Heideggerian question of the structure and givenness of time in the light of the concrete life of Dasein as well as the most important results of contemporary mathematical research.

In his essay "New Research on the Dialectical Structure of Mathematics," first published in 1939, Lautman develops the problem of the *structure* and *genesis* of mathematical objectivities, employing "certain essential distinctions in the philosophy of Heidegger" to demonstrate a specific kind of genesis of mathematical theories in what Lautman calls a "dialectic" that governs their constitutive structures as well as its concrete realization in practice. Here, Lautman (like Becker) refuses to locate the origin of mathematical objectivities and effective theories in a timeless realm of pure being, instead conceiving of the problem of the genesis of mathematical objects as intimately connected with the question of the givenness and structure of time itself. He reaches the conclusion that the capability of mathematics in understanding and influencing the physical world, and hence its application to the temporality determined by the phenomena of physical nature, must be understood on the basis of a more primary

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[12] Becker here relies primarily on the conception of time that Heidegger had expounded in his lecture on "Time" given to the Marburg theological faculty on July 25, 1924. As Heidegger suggests there and Becker emphasizes, authentic time cannot be conceived as primarily the outcome of a measuring or numbering, but must instead be grasped as a "coming-back" to "what is constantly the same unique instance," namely the "how of care" in which I "linger." In this way, time's running-forward into the future is not to be understood as an indifferent stretching-out or becoming longer; in fact, time originally has "no length in general" but is rather to be understood as containing "all time" within itself, including the very structure of its running-ahead, in the form of "momentariness" [Jeweiligkeit]. This authentic temporality, grounded in the original structure of Dasein's reflexivity, is to be sharply distinguished from the "non-historical" temporality typical of Dasein in its inauthentic everydayness; this inauthentic temporality, by contrast, is determined by measurement and by the clock. Its basic pattern, by contrast with the authentic temporality of Dasein, is set by the cycles of nature and the interpretation of processes in what is conceived as the natural world; thus, by contrast with the determination of authentic, "historical" time, determined by a momentariness that is in each case unique, the basic characteristic of this "natural" time is the possibility of the recurrence of the same.

and original order of genesis, one which also yields an original, pre-natural structure of time. This original time, for Lautman (as for Becker), is grounded in the reflexive and ec-static structure of Dasein, according to which Dasein is originally "transcendent" in that it exceeds itself and in a certain sense "surpasses" beings in the direction of its always presupposed, if typically inexplicit, fore-understanding of Being itself.

Lautman's 1939 work develops the thesis of his 1938 dissertation, according to which concrete mathematical theories develop a series of "ideal relations" of a "dialectic abstract and superior to mathematics." In particular, Lautman understands abstract "dialectical" ideas as the development of the possibility of relations between what he calls (by contrast) pairs of *notions*: these are pairs such as those of "whole and part, situational properties and intrinsic properties, basic domains and the entities defined on these domains, formal systems and their realization, etc." (p. 204). The dialectical ideas that pose these relations do not presuppose the existence of specific mathematical domains or objects. Rather, they operate, in the course of mathematical research, essentially as "problems" or "posed questions" that provide the *occasion* for inquiry into specific mathematical existents. In reference to differing specific mathematical theories such as, for instance, the theory of sets or (in a different way) real analysis, the dialectical relationship of whole and part may be seen as posing a general *problem* which is to be resolved differently in each domain, on the basis of concrete mathematical research, and thereby partially determines the kind and structure of entities which may be seen as existing in that particular domain. Thus general problems such as the problem of the relationship of formal theories of proof to actual mathematical results, the relationship of whole to part, and (especially) the relationship of continuity and discontinuity pose conditions under which they are resolved concretely, in different ways, in specific mathematical theories; at the same time, the development of the specific theories in terms of the particular kinds of structures and entities said to exist therein points back to the general problem and articulates its own more general structure.

The problem, here, thus has a priority over its particular solutions, and cannot be reduced to them. According to Lautman, this priority is not that of an ideality existent in itself prior to its incarnation in a specific domain, but rather that of the kind of problematic "advent of notions relative to the concrete within an analysis of the Ideas." In particular, it is only in developing the actual structure and configuration of particular concrete domains, that the actual meaning of the governing Ideas is worked out; here the concrete development of particular domains does not, moreover, exhaust the general problem but rather, typically, suggests new questions and problems in other concrete domains which are also to be related to the same general dialectical structure. Lautman sees this dynamic as structurally comparable to the analysis of the concrete structure of the factical disclosure of being undertaken by Heidegger (p. 200). In particular, here, as for Heidegger, the method of analysis depends, upon the possibility of the prior posing of a *question* and on the "prior delimitation" that this involves. This need not, as Lautman emphasizes, involve knowledge of the essence of the thing asked about but is rather based in what Heidegger calls a "pre-ontological" understanding. Like the posing of ontological questions on the basis of this "pre-ontological" understanding which first makes it possible, according to Heidegger, to interrogate specific beings as to their being, the posing of the dialectical questions is not separable from the questioning of the specific, concrete, ontic beings that are involved in each case. Rather, as for Heidegger, with disclosure of the superior, "dialectical" (or "ontological") truth, the

concrete structure of (ontic) beings is inherently *co*-disclosed, in particular with respect to the determination of the factual existence of the domains or regions in which they are categorically structured. In the analysis of the structure of mathematical theory, there is thus an anteriority of the global dialectical relationships "incarnated" in it to the specific theory; the priority of the dialectic is specifically "that of 'concern' or the 'question' with respect to the response." (p. 204)

Dialectical Idea, in this sense, "govern" the "intrinsic reality" of mathematical objects (p. 199). and it can even be said, using the Platonic terminology, that the reality of the mathematical objects, as concretely demonstrated in mathematical research, thus resides in their "participation" in the dialectical ideas. But as Lautman emphasizes, this sense of "participation" is quite at odds with the way Plato's conception of participation is typically understood; in particular, whereas participation is often understood as that of an ideal model to objects which in some respect copy them, here the Ideas are understood "in the true Platonic sense of the term" as the "structural schemas according to which the effective theories are organized." (p. 199). What is at issue here is not a "cosmological sense" of the relationship between ideas and their concrete realization such as is developed, for instance, in the *Timeaus*. According to such a sense, which is fundamentally understood by reference to the concept of creation as forming or shaping, the realization of the ideas in concrete reality depends on their capacity to impose law and structure on an otherwise undifferentiated matter, itself knowable only (as Plato in fact suggests) by a kind of "bastard reasoning" or "natural revelation." (p. 199) By contrast with this "cosmological sense" of the relationship between ideas and particulars, it is essential in the case of mathematical objectivity to understand the relationship between the dialectical ideas and the particular mathematical objects as a "cut [which] cannot in fact be envisaged," a kind of "mode of emanation" from dialectics to mathematics that does not in any way presuppose the "contingent imposition of a Matter heterogeneous to the Ideas." (pp. 199-200)

In the relationship between the dialectical ideas and the particular mathematical objects, there is thus a twofold relation of priority: although problems precede their concrete solutions as questions more generally precede their answers, it is of the essence of the articulation of the concrete domains of existence that it be possible only on the basis of a prior possibility of posing the questions which receive (partial) solutions therein.   The question of the determinate ontic structure of a particular entity thus always has reference back to the level of an ontological determination on which the question of its being can be posed. According to Lautman, this structural configuration is analogous or homologous to the structural "transcendence," which is, according to Heidegger, constitutive of Dasein. In particular, Dasein exhibits an "ontico-ontological" priority with respect to the fundamental difference between being and beings; because of this double priority, Dasein is both an entity with a privileged possibility of questioning about being and is structurally (ontico-ontologically) determined on this basis of this very possibility of questioning. This theory of the transcendence of Dasein is most completely developed, in close connection with the fundamental-ontological analysis of *Being and Time*, in Heidegger's 1928 essay "Vom Wesen des Grundes," upon which Lautman largely relies. In the essay, Heidegger specifies the "transcendence" of Dasein as consisting in the particular way in which Dasein, in relating to beings in its concrete, factical life, is always implicitly guided by an ontologically prior "understanding of the being of beings" in the sense of "the constitution of being: what something is and how it is". Heidegger

specifies this relationship of this "pre-ontological" understanding to beings by drawing a distinction between two kinds of truth: whereas "ontic" truth is the pre-predicative manifestness of beings, including their availability to specific acts of predication and judgment, this "ontic" truth of beings is itself preceded by an "ontological truth" which first makes possible this manifestness itself. This "ontological truth" is further specified (p. 130) as the "transcendental grounding" of things through the pre-ontological understanding of Being; it is the question of such grounding that is ultimately specified in the formulation of "why" questions such as the question "why [there is] something at all rather than nothing?" (p. 130).

Thus, whereas "ontic" truth always concerns beings as they are manifest, Heidegger understands this manifestness as possible only on the basis of a prior unveiledness of being itself which is already involved in the posing of every general question about beings and is guided by this inexplicit, pre-ontological understanding of being, itself structurally rooted in the disclosive structure of Dasein:

> The possible levels and variations of ontological truth in the broader sense at the same time betray the wealth of originary truth lying at the ground of all ontic truth. Unconcealment of being, however, is always truth of the being *of* beings, whether such beings are actual or not. Conversely, in the unconcealment of beings there already lies in each case an unconcealment of their being. Ontic and ontological truth each concern, in different ways, *beings in* their being, and *being of* beings. They belong essentially together on the grounds of their relation to the *distinction between being and beings* (ontological difference). The essence of truth in general, which is thus necessarily forked in terms of the ontic and the ontological, is possible only together with the iruption of this distinction. And if what is distinctive about Dasein indeed lies in the fact that in understanding being it comports itself toward beings, then *that* potential for distinguishing in which the ontological difference becomes factical must have sunk the roots of its own possibility in the ground of the essence of Dasein. By way of anticipation, we shall call this ground of the ontological difference the *transcendence* of Dasein. (pp. 105-106)

This means, as Lautman points out, that an explicit, conceptual understanding of being requires an analysis of this prior structure of the disclosure of being itself; in this analysis, the being of entities is marked off and fixed in general conceptual terms, "projected in general," and "expressly been made thematic and problematic." A primary moment of this projection is, as Lautman points out, the fixation of problems in the form of questions about entities; indeed, as Lautman emphasizes, according to Heidegger a determinate and essential moment of this process is the determinate "projection of the ontological constitution of beings" whereby a specific domain or field of beings (such as, Heidegger says nature or history) is marked off by means of specifying "fundamental concepts" that subsequently make possible the "objectification" of beings in this domain and their treatment by scientific means. Specifically, in this determination of regions by means of the fixation of problems:

> …a same activity is therefore seen to divide in two, or rather act on two different planes: the constitution of the being of the entity, on the ontological plane, is inseparable from the determination, on the ontic plane, of the factual existence of a domain in which the objects of a

scientific knowledge receive life and matter. The concern to know the meaning of the essence of certain concepts is perhaps not primarily oriented toward the realizations of these concepts, but it turns out that the conceptual analysis necessarily succeeds in projecting, as an anticipation of the concept, the concrete notions in which it is realized or historicized. (p. 201)

It is in the analysis of this "projection" of being onto specific domains of beings by means of the fixation of determinate problems and questions that Lautman identifies the possibility of a "general theory of [the] acts…which, for us, are geneses" (p. 200) and hence provides the essential ontological structure at the basis of the existence of mathematical (as well as other) entities in their specific conceptual determinacy.

As Lautman points out, this structure of transcendence is, for Heidegger, at the root not only of the specific "projection" of domains of entities, but also of the phenomenon of "world" in general. On Heidegger's analysis, in particular, world is not simply a totality of extant entities but always to be understood as constitutively related to the structure of Dasein, and specifically to its transcendence or (what Heidegger, in "On the Essence of Ground," suggests is the same) its being-in-the-world. In this structure, world is that "toward which" or "for the sake of which" Dasein directs and ultimately comports itself (p. 121); thus Dasein surpasses beings as a whole *toward* the world; in this surpassing "world" does not refer to a totality of beings but to the "*how" of being* of beings as a whole; this reference is in a certain sense "prior" but remains specifiable only relationally, indeed "relative to human Dasein" (p. 112). There is thus a certain paradox involved in the place of Dasein in the world. Heidegger recounts versions of this paradox as they are suggested in Heraclitus and by Kant; according to Heidegger, what is "metaphysically essential" in all existing interpretations of *kosmos*, *mundus*, and world is the reference of each of these concepts to the interpretation of the relationship of human existence "to beings as a whole" (p. 121). In this structure, the world "belongs precisely to human Dasein, even though it embraces in its whole all beings, including Dasein". (p. 112); in this respect, the world is thus structurally based on the originally reflexive/paradoxical structure of Dasein itself. Indeed, more generally, the "selfhood" of Dasein itself depends on this original surpassing of Dasein by itself in the direction of world:

> World as a wholeness 'is' not a being, but that from out of which Dasein *gives itself the signification* of whatever beings it *is able* to comport itself toward in whatever way. That Dasein gives "itself" such signification from out of "its" world then means: In this coming toward itself from out of the world Dasein gives rise to itself [*zeitigt sich*] as a self, i.e., as a being entrusted with having *to be.* In the being of this being *what is at issue is its potentiality for being.* Dasein is in such a way that it exists *for the sake of itself.* If, however, it is a surpassing in the direction of world that first gives rise to selfhood, then world shows itself to be that for the sake of which Dasein exists." (p. 121)

As Lautman emphasizes in his own interpretation, this does not mean that the world exists only as a result of the factual existence or ontic activities of humans, for "it is not necessary that the sort of being we call human …exists factically. It can also *not* be." (p. 201). But on the level of properly ontological

genesis, it points, according to Lautman, to the specific relationship between logical and creative determination at the root of every possibility of the grounding of entities by means of their rational explanation *or* their creative foundation. In particular, Heidegger understands Leibniz's principle of sufficient reason as the specific *interpretation*, within a specific and delimited understanding of "being in general", of the underlying unitary conjunction of truth, ground, and transcendence; this specific interpretation, according to Heidegger, finds further application within essentially the same idea of "being in general" in Kant's determination of the reality of transcendental truth "via the unity of time, imagination, and the "I think."".   But when this original unity of being and grounding is grasped on the more originary basis of the structural transcendence of Dasein, what appears behind and at the basis of the application of the principle of sufficient reason to the rational determination of entities is a more basic creative activity of founding, which itself can be understood as a structurally original freedom. Thus, by contrast with the principle of sufficient reason itself, which Heidegger understands, according to Lautman, simply as a "logical principle," there is a deeper transcendental principle implicit in every posing of the question of "why" and specifically involved in the "rather than…" of the questions: "why something exists rather than another thing"; "why something rather than nothing":

> For Heidegger it is a transcendental principle that the determination of the entity necessarily relies on a creative freedom rooted in the ontological constitution of the being that determines. It is thus that [rational] grounding gives rise to the formation of a project of the World, in which the creative freedom of the founding power is asserted (in the dual sense of founding and foundation). (p. 203).

According to Lautman, this freedom is not ontic or empirical freedom but rather a freedom of Dasein that is structural, and thereby points back to underlying temporality itself (p. 203). The structural configuration that here indicates a deeper structure of ontico-ontological genesis at the root of both the specific constitution of particular material domains and the possibility of Dasein's possible disclosure of them is quite general, and indeed can be seen as a structural-genetic precondition for the determinate being of beings in any number of domains. According to Lautman, this account of ideal genesis can, moreover, be separated at least to some extent from Heidegger's own preconceptions linking it to the specific projects of a "human" Dasein. Thus, although Heidegger himself assuredly thinks of the genesis of the "project of the World" as founded specifically in the idea of "human" reality, it is nevertheless, Lautman suggests, possible to read his genetic conception as having the more general significance of "a genesis of notions relating to the entity, within the analysis of Ideas relating to Being" that is characteristic of the determinate ontico-ontological ideal constitution of entities *in general* and bears no necessary reference to "human" being or anything specifically characteristic of it. (p. 202).

Understood in this way, the ontico-ontological account of has general bearing on the constitution, not only of mathematical reality, but of other "domains", including that of "nature", as well. For example, even beyond Heidegger's own "anthropological preoccupations", one can and should read this account of ideal genesis as bearing (as Heidegger himself suggests) on the basic concepts of *physical* science, such as "space, locus, time, movement, mass, force, and velocity." (p. 202) Indeed, by looking beyond the "anthropological" determination of Heidegger's project in terms of specifically "human" reality, it is

more generally possible to see the real implications of his structural account of genesis as bringing out the implications of mathematical philosophy for "metaphysics in general:"

> Whereas for all the questions that do not come out of the anthropology, Heidegger's indications remain, despite everything, very brief, one can, in regards to the relation between the Dialectic and Mathematics, follow the mechanics of this operation closely in which the analysis of Ideas is extended in effective creation, in which the virtual is transformed into the real. Mathematics thus plays with respect to the other domains of incarnation, physical reality, social reality, human reality, the role of model in which the way that things come into existence is observed. (p. 203)

According to Lautman, the specific kind of relationship, characteristic of mathematical philosophy, that exists between the dialectical ideas and particular domains of existence is illustrated in an exemplary fashion by the metamathematical results of Godel and those who immediately followed him, which put an end to the debate of the 1920s between intuitionists and formalists, or at least situated it on very different ground. Near the conclusion of his principal thesis of 1938, "Essay on the Notions of Structure and Existence in Mathematics," making reference both to Godel's 1931 incompleteness result and to the proof of the consistency of Peano Arithmetic, by means of transfinite induction on the length of formulas, achieved by Gentzen in 1936, Lautman suggests that the particular situation of philosophical analysis with respect to mathematical problems is illuminated by both results. In particular, both Godel's limitative result, which shows that there can be no proof of the consistency of a theory by means of that theory itself, and Gentzen's positive one, which proves the consistency of arithmetic but only, as Gentzen himself says, by means that no longer belong to arithmetic itself, bear witness to the "exigency" of the logical problem of consistency with respect to any particular theory. This marks the distinctive status of a "metamathematical" inquiry into the nature of mathematical knowledge which essentially depends on, and accommodates itself to, logical results without being simply reducible to them. It is possible, in particular, to see "how the problem of consistency makes sense" without yet being able to resolve it by mathematical means. It is within such an "extra-mathematical intuition of the exigency of a logical problem" (pp. 188-189) that the whole foundationalist debate of the 1920s has essentially taken place, and it is only by drawing on it that Godel's results were able to transform the problematic situation and place it on new grounds.

More generally, with respect to problems such as that of "the relation between the whole and the part, of the reduction of extrinsic properties to intrinsic properties," or "of the ascent towards completion," progress in general depends not simply on the application of pre-existing logical schemas or regulative logical conceptions (such as the ones governing the competing approaches of formalism and intutionism in the 1920s) to already-defined domains but rather on the constitution of "new schemas of genesis" within the concrete progress of mathematics itself. Here, the philosopher's role is "neither to extract the laws, nor to envisage a future evolution," but only to "[become] aware of the logical drama which is played out within the theories." (p. 189). In this awareness, the philosopher does not and cannot identify general *a priori* conditions for possible theories or existents; indeed, the only *a priori* element here is to be found in the specific anteriority of the general problems to their solutions in particular

domains. This is, moreover, an *a priori* only in a "purely relative sense"; it consists only in the "possibility of experiencing a mode of connection between two ideas and describing this concern phenomenologically independent of the fact that the connection sought after may, or may not, be carried out." (p. 189) Some of these connections, and the mode of concern that is correlative to them as the basis for the posing of the question of their particular possibility, are visible in the historical concerns of philosophers, for instance with the relationships between the "same and the other, the whole and the part, the continuous and the discontinuous, essence and existence." (p. 189) But the mathematician's activity has an equally significant role, according to Lautman, in giving rise to new problems that have not yet been abstractly formulated. In this twofold enterprise, the task is thus not to demonstrate the applicability of classical logical or metaphysical problems *within* mathematical theories, but rather to grasp the structure of such theories "globally in order to identify the logical problem that happens to be both defined and resolved" by its existence. (p. 189). This is a peculiar experience of thought, according to Lautman, equally characteristic of the capacity of the intelligence to create as of its capacity to understand. In it,

> Beyond the temporal conditions of mathematical activity, but within the very bosom of this activity, appear the contours of an ideal reality that is governing with respect to a mathematical matter which it animates, and which however, without that matter, could not reveal all the richness of its formative power. (p. 190)

Finally, Lautman suggests that this particular experience of exigency, by means of which general philosophical problems communicate with the particular constraints of specific mathematical domains to illuminate the "contours" of such a superior reality, can be witnessed in the late Plato's understanding of the dynamical genesis of Ideas and numbers, as it is reconstructed by the "most authoritative" contemporary interpretations of Platonism:

> All modern Plato commentators … insist on the fact that the Ideas are not immobile and irreducible essences of an intelligible world, but that they are related to each other according to the schemas of a superior dialectic that presides over their arrival. The work of Robin, Stenzel and Becker has in this regard brought considerable clarity to the governing role of Ideas-numbers which concerns as much the becoming of numbers as that of Ideas. The One and the Dyad generate Ideas-numbers by a successively repeated process of division of the Unit into two new units. The Ideas-numbers are thus presented as geometric schemas of the combinations of units, amenable to constituting arithmetic numbers as well as Ideas in the ordinary sense. (p. 190)

Following Stenzel and Becker, Lautman suggests that the diaeretic "schemas of division" of Ideas in the *Sophist* can themselves be traced, in their logical structure, to the schemas of the "combination of units" that are also responsible for the generation of the Ideal numbers. Both are then genetically dependent upon a kind of "metamathematics" which unfolds a time of generation that, though it is not "in the time of the created world" is nevertheless, just as much, ordered according to anteriority and posteriority. This ordering according to anteriority and posteriority is equally determinative, and even in the same

sense, with respect to essences quite generally as with respect to numbers themselves. Indeed, following a suggestion by Stenzel, Lautman suggests that this is the significance of Aristotle's claim that (EN 1.4) the Platonists did not admit the ideas *of* numbers: since the ideal-numbers are *already* the principle of the determination of essences as anterior and posterior (i.e. as before and after), there is not (nor can there be) a *further* principle of the division of essences that is prior to or superior to this numerical division itself. In this impossibility of equipping the metamathematics of the ideal-numerical principles of anteriority and posteriority with *another* determination (a "metametamathematics", so to speak), we witness once again, according to Lautman, the necessity of pursuing the dialectic in which the mathematical problems and the ideal relations communicate with and articulate one another. In particular, in such a dialectic, and only in it, are to be found the problematic conditions and the possibility of mutual illumination in which the more original structures constitutive of anteriority and posteriority *as such* – and hence of time and genesis, in an original sense – can be brought to light.

IV

As we have seen, beyond the general fact that both seek to illuminate the problems of mathematical ontology by reference to Heidegger's development of the "fundamental ontology" of Dasein, there are also several more specific points of agreement between Becker and Lautman. Both find in the specific reflexive structure of Dasein by which it is characterized as both ontic and ontological the conditions for the possibility of an illumination of the mode of being of mathematical objects that also illuminates, at the same time, the conditions of their possible discovery or disclosedness in factical life. In both cases, these conditions are intimately linked to the conditions for the possible disclosive openness of the world as such, as these are themselves related to the constitutive ideas of totality, infinity, and the process toward an (actually or potentially infinite) horizon. Both Becker and Lautman consider how these constitutive ideas are linked and how their linkage is illuminated by the developments following from Cantor's creation of transfinite set theory and the new kind of availability of the actual-infinite to mathematical cognition that it makes possible. Both further suggest that this provides an important clue to the original constitution of *time*. And finally, both suggest that this interconnection can itself be seen as structurally anticipated or actually developed in the methodological and thematic views of the late Plato, developed under the condition of the general question of the *apeiron* as it presented itself to him, and especially with regard to what appears there to be the description of a dynamic genesis of ideas and numbers that is structurally prior to empirical or worldly time but nevertheless constitutive of it.

As pointing to aspects of the structurally underlying connection between the specific structure of Dasein and that of time, each of these suggestions may clearly be considered significant for the broader ontological problematic within which Heidegger already situates the analytic of Dasein in *Being and Time*, that of the relation between being and time themselves. The suggestions that Becker and Lautman both make in the course of their investigations of the specific problems of mathematical ontology point, in particular, to the possibility of an ontological interrogation of the actual *being* of time as it is, and insofar as it is, *mathematically* graspable and thinkable. Such an interrogation is, as I have

suggested, plausibly not only requisite for a clarification of the structural possibility of what Heidegger calls "world-time" but in fact is positively entailed in the very idea of a realist conception of time itself; here, it is plausible that the mathematical form of time does not attach to it externally or only as an accidental determinant of what is in principle non-mathematical; rather, both Becker and Lautman point to the way in which the very structure of serial order, or of anteriority and posteriority as such, *already* involve the mathematical development of number in the "ontological" problematic of being and time. In this structure, on almost any conception of it, the idea of the infinite is already involved, and its interpretation according to one or another figure will thereby determine the kind of availability with which it can be characterized in the course of an application of it to the thinking of time in itself. This suggests not only an illuminating connection between the specific developments of the idea and problem of the actual-infinite in the wake of Cantor and the "Heideggerian" problematic, but also a more basic link of both to the illumination of the structural conditions characteristic of given time.

Nevertheless, there are also important differences between the specific conclusions that the two philosophers draw about the nature of these connections in particular. First, as we saw above, Becker's concern with the concrete situation of mathematical research led him to take up the question of the phenomenological foundation of mathematical objectivities in terms of the foundational debate, then current, between Hilbert's formalism and Brouwer's intuitionism. Because of the requirement, in order for mathematical existence actually to be demonstrated in concrete experience, of the actual "executability of factical syntaxes" and combinations of signs, Becker judged the hermeneutic inquiry to decide the debate, on the basis of the actual structure of concrete life, in favor of Brouwer's intuitionism, provided (of course) that the intuitionist position not be understood simply as finitist, but as including the possibility of an actual givenness of infinite and transfinite structures in concrete life. More generally, Becker saw this as deciding in favor of an "anthropological" orientation toward mathematical existence, one that, in connection with the decisive link that Becker demonstrates between the structure of the infinite and that of time, suggests that time itself must be understood as given ultimately through a basically "human" structure of life and experience. By contrast with this, Lautman expresses skepticism about the prevalence of "anthropological" determinants in Heidegger's own thinking about transcendence and ontology, and positively suggests the possibility of a very general ontic-ontological analysis of the ideal conditions for the articulate existence of intelligible entities quite generally that owes nothing to any specifically "human" determination. Here, what is most significant in showing the underlying structure of disclosure and world is exactly not the *actual* "executability of factical syntaxes" but rather, as Lautman says, "the possibility of experiencing the concern of a mode of connection between two ideas and describing this concern phenomenologically *independent of the fact that the connection sought after may, or may not be carried out*." And if Lautman can thereby refer to the structure of a superior dialectic of ideas that is "governing" with respect to mathematical practice although nevertheless always concretely developed within it, he does not do so by referring this practice to any methodological criterion – whether it be the formalist idea of the unlimited continuation of non-contradictory inference, or the intuitionist limitative strictures on proof – imposed from outside it by a hermeneutic condition that is simply exterior to that practice itself. Rather he aims to develop the structure of disclosure as it actually takes place within the practice of mathematicians, and finds there a remarkable *confirmation* of the ontic-ontological structure independently developed by Heidegger.

Second, Becker and Lautman conceive of the *relationship* between what both see as Plato's understanding of the ideal genesis and Aristotle's own account of time and number very differently. In particular, whereas Becker conceives of Plato's picture as articulating a kind of formal and structural *precedent* to Aristotle's development of time as the measured "number" of motion, and in particular of the hylomorphism involved in its conception of the measurement of time as depending on the external division of the continuum into discrete "now" moments, Lautman points to the inherence in Plato's dialectic of a very different kind of structure of genesis. This is not a structure of potentiality that precedes the actuality of the *measurement* of the now, but rather a fully *actual* conjoint ideal generation of the ideas and numbers in the dialectic that presides over mathematical practice. Here, the fixation of formal relations is to be sharply distinguished, as Lautman emphasizes, from any imposition of formal structure on a matter simply exterior to it. The time of genesis that it articulates is anterior to that of actual measurement and counting, not as the potential precedes the actual, but as the *actual* precedence of the problem over its concrete development, or of the inexplicit pre-understanding that provides the actual hermeneutic basis for ontic-ontological questioning.

The two differences are related on the level of the determinations of the ultimate structure of *time* that they both suggest. In particular, if Becker can conclude from his hermeneutic of mathematical practice that intuitionism is to be preferred over formalism as a description of its temporal facticity, this is only because he sees the applicable temporal structure as essentially that of the "endless becoming" of the infinite as *Aristotle* conceives of it. The actual structure of temporality that is involved in the development of the structure of the transfinite hierarchy, and for which Becker finds a concrete motivation in the actual possibility of the continuation of reflection up to and through transfinite levels, is not either that of a fixed, static structure progressively disclosed or an infinite development by means of a static repetition of a rule (what Becker identifies with the Hegelian bad infinity). Rather it is to be understood, according to Becker, as a constant becoming with a certain character of freedom, and this is what verifies the greater relevance of the intuitionist picture and with it, the more general "anthropological" orientation as opposed to formalism. It is true that the successive progression to new transfinite levels corresponding to Cantor's second formation principle, which Becker sees as factically motivated in the structure of the possible levels of reflection, have no direct parallel in Aristotle's picture of time and becoming. And it also not that the kind of becoming they represent is to be thought, according to Becker, as simply the repetition of a rule in the unchanging medium of the *aei on*. But even so, their development is to be modelled, according to Becker, as an infinitely extensible development *in potentio* without possible completion, having its basis in the reflexive structure of finite Dasein. By contrast with this, Lautman points, as we have seen, to a "superior dialectic" of Ideas that is incarnated in mathematical practice but not exhausted by it. This dialectic is explicitly modelled on the dialectic of Ideas in Plato, and particularly on his developments of it in the later dialogues. These developments witness, as we have already seen and will confirm further in the next section, a profound engagement in the late Plato with the problems of the structure of the *apeiron* as it is available to thought, and with it, the question of its *actual figuring* in temporal becoming. Here, the figuring of the apeiron in being and in thought is not limited to its potential development in always-finite fragments; it is rather insistent on

the level of the "posed problem" of the real constraint it places on the interlinked structures of thought and beings.

In Becker and Lautman's two developments of the implications of the factical hermeneutics of mathematical practice, we thus see two very different pictures of underlying temporality apparently confirmed. On the one hand (Becker) we have an unlimited structure of possible continuance that confirms an "anthropological" picture of the ultimate temporal basis of mathematical reality; while on the other we have an immediate insistence of the *aperion* – as an inherent component of the various posed problems of the one and the many, the finite and the infinite, and the continuous and the discontnous – as articulating the real structure of a prior temporality of ideal genesis that owes nothing (according to Lautman) to any specifically anthropological determination.

Can a basis for resolving this difference be found in the actual development of mathematical and metamathematical reflection? Without a doubt, the metamathematical results achieved since the heyday of the intuitionist-formalist debate in the 1920s that bear the deepest implications for both pictures are Godel's two incompleteness theorems and the closely related results of Turing about computability and undecidability. Since these results themselves turn closely on issues about the infinite, the transfinite, and the form of their availability to human thought, it is worth considering whether they bear implications for the premises of Becker and Lautman's different conceptions of the nature of the infinite as it figures in mathematical thought and practice. And since they also (as we have seen in chapter 5) clarify the very structure of mathematical knowledge, truth, and proof, they may reasonably be thought to bear closely, as well, on the issues of mathematical ontology that both take up.

As is familiar, Godel's first incompleteness theorem (in the slightly strengthened form due to Rosser) establishes that for any consistent formal theory of a certain degree of expressive power, there is a sentence (the so-called Godel sentence) which is undecidable in the sense that neither it nor its negation is provable in the system. The second incompleteness theorem shows that no such system can, by itself, prove a statement of its own consistency. Turing, considering the problem of finding mechanistic decision procedures for solving classes of arithmetic problems, showed that there are certain well-specified problem classes for which there is no decision procedure, thereby showing that there are (many) *uncomputable* real numbers – numbers whose decimal expansions are not determined by any finitely specifiabile algorithm – and that solutions to certain well-specified problems are uncomputable in this sense. Collectively, these results (and especially Godel's second theorem) have often been taken to defeat Hilbert's formalist program by showing the impossibility of giving an internal "consistency proof" for formal systems of the kind Hilbert sought and proposed as a precondition for its success. On the other hand, though, these results also give little comfort to intuitionism in its classical forms. To begin with, they cannot even be stated within *finitist* forms of intuitionism. More seriously, the forms of incompleteness and undecidability they demonstrate do not clearly invite description in terms of intuitive presentations, "free choice" sequences, and the like. Godel himself, as is well known, saw his results as evidence for an (unorthodox) Platonism, one that he also saw, in the last years of his life, as consonant with at least some aspects of Husserl's phenomenology. Most generally, although the

phenomena of undecidability and uncompleteness do not resolve these debates in favor of any single, well-understood position on the reality and givenness of mathematical objects and truths, it appears that they may witness a completely different and *sui generis* phenomenon of givenness, one to which we may have to look to understand the phenomenon of the givenness of mathematical truth in general more clearly. In seeing the truth of the Godel sentence for a particular formal system, one grasps on the basis of a proof-theoretical argument what may be seen as an arithmetic truth that is *provably* beyond the capacities of that system to demonstrate, assuming it is consistent. As we have already seen in chapter 5, this suggests an inherent formal limit of the capacities of finitely specifiable systems, in relation to the irreducible "inexhaustibility" of mathematical truths that no of them is thus able to capture in its totality. Whether this is understood as pointing to an essential limitation of human cognition itself, or rather to the somewhat mysterious inherence within it of a form or possibility of "insight" going essentially beyond finitely specifiable capacities, it is clear that it articulates in a fundamental way what can subsequently be said about the constitutive forms and ultimate structure of the possibility that truths are given to an essentially finite intellect.

Writing in 1927, Becker in *Mathematische Existenz* (by contrast with Lautman, in his works of the late 1930s) did not have the benefit of Godel's or Turing's results. Nevertheless it is noteworthy that at least *one* part of his argument there appears to receive significant further support from them. As we saw, Becker argues that the development of the transfinite hierarchy is marked by an essential character of openness or freedom, which Becker sees as grounded in Dasein's "historical" temporality itself; a consequence of this freedom is that successive stages of the transfinite hierarchy are, although determined regularly by what has come before, also in a certain sense unforeseeable in advance. In fact, the phenomena of the undecidability and incompleteness of formal theories, on which Godel's and Turing's results turn, are *intimately* related to this phenomenon of the indeterminacy (or irregularity) of the successive stages of the transfinite hierarchy, to which Becker appeals at a decisive point. As Godel himself suggested in a footnote to his 1931 paper:

> The true source of the incompleteness attaching to all formal systems of mathematics, is to be found–as will be shown in Part II of this essay–in the fact that the formation of ever higher types can be continued into the transfinite ... whereas in every formal system at most denumerably many types occur. … Namely, one can show that the undecidable sentences which have been constructed here always become decidable through adjunction of sufficiently high types (e.g. of the type ω to the system **P**). A similar result holds for the axiom systems of set theory. [13]

Godel here suggests that incompleteness results, in a fundamental way, from the *possibility* of forming the extended transfinite hierarchy, which relentlessly and infinitely outstrips the possibilities for expression given in any well-defined formal system, given that every such system can formulate only *countably* (denumerably) many possible expressions. For each such system, we can (as Godel showed) always constructively generate an undecidable sentence of the Godel type; if we then add the undecidable sentence to the system as an axiom, we produce a new system, but one for which we can then, once again, generate a new undecidable sentence. We can continue the process through indefinitely many finite levels; and we can indeed, as Godel says, continue it into the transfinite, adding

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[13] Gödel (1931), footnote 48a, pp. 28-29

(for instance) a sentence which expresses the limit of this process at level ω. But at each stage, there will be further undecidabilities, and it is not possible to summarize the *whole* process in any finitely expressible form which would also be capable of telling us how to iterate it at each stage.

The suggestion of a link between the character of the transfinite progression and the incompleteness of formal systems which Godel makes in the footnote has subsequently been developed in two rather different ways. The first is the investigation of the consequences of iterating the development of successive formal systems by means of what have been called "reflection principles"; such a principle for a particular formal system is, for example, one that asserts that all of its consequences are true, or an assertion of the consistency statement for that formal system itself. The inclusion of such a principle results in the production of a new system which can prove more than the original system; the question thus arises whether it is possible, by means of such a progression of theories through transfinite levels, to prove all arithmetic truths. Feferman (1962) has shown that there is in fact a certain kind of completeness that can be achieved by a suitable (transfinite) iteration of reflection principles in this way: *all* elementary arithmetic truths can be proved through an appropriate iteration of theories. However, because the need to represent the theories involved at each stage introduces an aspect of intensionality into the iterative process, there is no unique way to specify in advance the form that such an appropriate iteration will take. This leaves very much open, for actual mathematical practice, the question that is at stake between the mechanist and the non-mechanist interpretations of Godel's results (discussed above in chapter 5): namely, whether successive insight into the truth of Godel sentences for particular theories witnesses, in each case, only the finitely specifiable capacities of the determinate formal system that *we*, ourselves, embody, or whether there is a kind of extra-mechanical insight operative in it that exceeds the capacities of any such system.

The second, rather different, development of Godel's suggestion in the footnote (and in particular its last sentence) relates to the profound set-theoretical problem of the continuum hypothesis, which Cantor already posed in his lifetime and was vehemently pursued over the next several decades. Through Godel's own results in 1940 and Cohen's in 1963, we now know that the continuum hypothesis is *independent* of the axioms of ZFC set theory: that is, neither it nor its negation can be proven by its means. This formally demonstrable recalcitrance of the CH to proof or refutation on the basis of natural axioms has often been taken to demonstrate that the CH is either (in some sense) ill-defined or simply undecidable, some researchers have continued to pursue the idea that adjoining new axioms to ZFC may suffice to resolve its status. In particular, there is some reason to believe that the adjunction of certain "large cardinal axioms" – axioms asserting the existence of very large transfinite cardinals – may provide a basis for settling the CH in the positive or negative. Nevertheless, despite some promising initial results, the "large cardinal" axioms lack the degree of intuitive plausibility characteristic of the existing ZFC axioms, and none has yet been shown to be able to resolve the question of the CH determinately one way or the other.

In these ways, both Godel's results themselves and the further development of their consequences appear to confirm the suggestion of a deep connection between the development of the transfinite hierarchy and an underlying phenomenon of undecidability or essential incompleteness which, as we

have seen, Becker already made on the basis of the partial results of set theory at his time. As Becker already suggested, they verify that the iterative development of reflective mathematical theorizing can be continued, in principle, indefinitely through the transfinite hierarchy, and that at no particular stage of this continuation does the phenomenon of undecidability completely subside. It is therefore apparently possible to speak, as Becker does, of an essential "freedom" involved in this development, whether it is conceived as the successive development of the hierarchy of ordinals themselves or as that of an ordered progression of formal theories. Moreover, the persistence of undecidability even given the repeated adjunction of reflection principles appears to parallel or confirm the link that Becker already draws between this "free" character of transfinite development and the actual structure of the stages of Dasein's concrete reflection on itself, which for Becker is grounded in the essential freedom of Dasein's "historical" temporality.

As we have seen, as well, Becker further connects the "free" and endlessly developing character of the transfinite sequence to the temporal basis that he sees, along intuitionist lines, as following from the necessary temporal form of a *human* agent at its actual and factical basis. The development of what cannot be predicated all at once and in advance by finite means must be actually undertaken, by a finite agent, in its own ongoing but always finite time. Thus the specific phenomenon of the "unpredictable" or "free" development of the transfinite hierarchy is thus seen as confirming the correctness of an intuitionist picture of mathematical objectivity as irreducibly given on the basis of its factical and concrete demonstration in a human life. For Becker, the analogy to the intuitionist "choice sequences" is, here, direct. Such sequences must essentially be developed step by step over time; this is a consequence, according to Becker, of "our time-boundedness." By contrast with a divine intellect which could survey them all at once and thus would avoid the necessity of such a sequential counting or enumeration in time, both the free choice sequence and the progression of ordinal numbers, including its tranfinite development, must be given "for us" only in their sequential and step-by-step development in time. This confirms, according to Becker, the relevance to the ontological problematic of temporality as the "exquisitely *human* moment of Dasein" which can thereby be seen as conditioning all mathematical existence in general.

Do the phenomena of incompleteness and undecidability, considered as indicative of the underlying structure of the temporality of mathematical thought in relation to the truths it discloses, then further verify this "anthropological" conception of the actual temporal basis of mathematical existence? In fact, they do not. This can be seen by considering once more the general implications of the undecidability results, this time in the specific form that they receive through Turing's proof of the unsolvability of the halting problem. As we have already seen, in fact, in chapter 5, Turing's proof of the limits of any finitely specifiable formal system with respect to the decision of mathematical questions does not depend on any formulation of the typical or representative capacities or abilities of a specifically human agent or thinker. It turns, rather, on the highly general concept of an *effective procedure*, or one that can be specified finitely and can be guaranteed to terminate in a finite number of steps: although this idea certainly involves *a certain* conception of finitude, this is not a conception that depends in any sense on any essentially "anthropological" conception of specifically human capacities or faculties. In demonstrating the inherent limitations of effective procedures in this sense, Turing's result does not

witness a contingent limitation of the human (or any other) intellect, but (much more) something like the necessary and constitutive limitation of the very ideas of procedures and capacities themselves; and the truth of undecidability is thus shown to be inherent in the very structure of mathematical truth, insofar as it can be thought *at all*. It is relevant to this that, in the face of the phenomenon of undecidability, the contrast that was already drawn by Kant and which Becker repeats, that between a divine intellect that is situated outside time and the human intellect situated within it, no longer applies: for example, even the divine intellect would not be in a position to specify a decision procedure for an undecidable problem. Nor, relatedly, could such an intellect render the "maximal" ordinal W non-contradictory, or eliminate the contradiction inherent in the Russell set. If these aporeatic results of metamathematical reflection are taken seriously as pointing to real structural aspects of mathematical truth, they thus already point beyond the "anthropological" conclusion that Becker draws. Admittedly, it is always possible to reinstate the intuitionist solution, for example by simply denying, by means of finitist or limitative strictures, the formulability of the problems to which these results respond. But to do so is not to reckon with, but just to ignore, the constitutive and ideal reflection they perform and the specific underlying structures that are indicated on their basis.

Given the local terms of the intuitionist/formalist debate, Becker is probably forced to the specific conclusion he draws by his desire to avoid the formalist position, which on the one hand abstracts from actual mathematical practice and on the other conceives of mathematical objectivity in the (presumably) timeless form of abstract symbolic relations. But the overcoming of this debate on the level of proof theory through Godel's and Turing's results points the way to another way of conceiving of the relationship of time to mathematical truth, one which is neither the dependence of inquiry on time that the intuitionist centrally pronounces nor the timelessness of formal/syntactical relations envisaged by the formalist. In particular if the structural phenomenon of undecidability can indeed be taken seriously as indicative of a particular temporal structure, this is quite different than the temporal structure on which the intuitionist argument turns, that of "our" essential boundedness in time, but it is also not the simple exteriority to time envisaged by the formalist or the (banal) "Platonist." By contrast with both of these positions, the temporality of mathematical development enters here as an *essential* feature of mathematical truth; this temporality, which is *both* the temporality of constitution and of disclosure, is here characteristic of truth as such, and not simply its character as shown to or pursued by *us*. As I have already argued, in chapter 5 above, the demonstrated character of undecidability in relation to the phenomenon of the "inexhaustibility" of mathematical truth imposes the requirement of a stringent *realism* with respect to the structural characteristics of the manifestness of forms; in particular, the metamathematical results which verify the inherency of undecidability in the disclosure of mathematical truth themselves already (I argued there) formally indicate the inherent point of a structural Real that cannot be referred to any anthropologically based or simply intra-temporal production or creation. As I further suggested, the metaformal realism that is thereby indicated can itself be related to the "Heideggerian" ontological problematic, in that the structure of the formal indication of Being is itself homologous or actually identical to that of this metaformal reflection.

What is the result, then, if the metaformal realism I recommended there is applied to the problem of time as it is originally given? The answer appears to be that the demonstrated phenomena of

incompleteness and undecidability themselves can be seen as indicative of underlying constitutive structures of time and its givenness, where they are no longer simply negative and limitative, but rather also positively indicative results. Thus conceived, they unfold the implications of a constitutive infinitude that is given to thought as both the general medium in which (indifferently finite and infinite) number subsists and the general form of given time, and which is subject in its very structure to undecidability. In connection with the ontological problematic, this undecidability can be seen as articulating the original form in which the givenness of something like time becomes structurally possible; and this provides a dramatic alternative to the Aristotelian conception, which (as we have seen) develops the givenness of time rather on the basis of the presumptively finite form of its intra-temporal counting. With this, it is possible to break with the whole Aristotelian picture of the determinacy of time's givenness in measurement and the actually existent "now" as mere particular actualizations of an indeterminate potentiality, itself fully given in advance; rather than resulting from the exterior and potential application of numerical measurement to a matter originally and in itself indifferent to it, original time is rather here to be grasped as unfolded in the actual structure of number itself, as determined by and determining the ideas and paradoxes of the infinite, the punctual, and the continuous which are clarified within it by means of metaformal reflection.

This is the dialectical determination on which Lautman, in his own picture of mathematical discovery and truth, insists, and which he links to the suggestions of a prior genesis of the ideas that are visibly determinative for Plato's own conception in his later dialogues. If Lautman is prevented from drawing out all the consequences of this account of ideal genesis for the underlying structure of given time, such as it is both thinkable and real, in itself, it is because he focuses his attention on mathematical practice and does not link the procedures of "mathematical philosophy" *directly* and in detail to the ontological problematic of being and time as I have suggested here. But if the ontological problematic indeed itself can be significantly developed, as I have suggested, on the methodological basis of a metaformal realism that furthers its questions by means of a reflective interrogation of the givenness of forms, then the relationships of the superior dialectic that Lautman suggests indeed bear direct and significant implications for its indication of the underlying character of given time. The constitiutive problematics of this indication may then be seen as formally underpinning any specific idea of the givenness of being to thought as such, and as especially inhering in the systematic developments of the form of this givenness that are characteristic of this relationship as it is has been thought in the history that Heidegger designates as that of Western "metaphysics." It is to this underlying exigency of this problematic of being, becoming, and the givenness of time to thought, as it is determinately thought, prior to its Aristotelian subjection to the anthropological rubrics of intra-temporal finite measurement, in Plato's own late conception of an ideal genesis that unfolds the prior temporality of both (supersensible) ideality and (sensible) reality, that we now turn.

V

In the foregoing sections, I have developed the suggestion of a basic structural link between the givennesss of time and the structure of number, including its inclusion of the specifically paradoxical structure of the infinite or *apeiron*. I have further suggested that the question of the structure of this original link is inherently involved in the development of the ontological problematic, at the point at which it ventures to ask about the constitutive form of given time. The further development of this question, in light of historical and contemporary developments of mathematical and ideal reflection, provides the basis for a critical deconstruction or actual alternative to the criteriological or anthropological conception of counted time that determines the form in which the givenness of time is thought in philosophers from Aristotle to Kant. The alternative is posed, in large part, by developing the implications of the original structural paradoxes of becoming and its availability to thought that are foreclosed (as Derrida suggests) or avoided in the Aristotelian conception of the infinite as the *dunemei on* and in the structure of essence and accidence that he draws from it. The problem of the being of the infinite and its link to the temporal structure of becoming in itself can then be retrieved both by means of the interpretation of the internal development of metamathematical or metalogical problematics, and also discerned at the historical foundation of the "metaphysical" interpretation of being as presence and of the mathematical/ideal as the *aei on.* In particular, as we have already seen reason to suspect, it can be discerned in thought of the late Plato, where the original problem of the paradoxical structural configuration time, becoming and the *apeiron* is (prior to and by contrast to its Aristotelian foreclosure) still alive as an actual and decisively determining problem of ontological research.

In particular, the paradoxes of the actual inherence of the *apeiron* appear in Plato's middle and later dialogues in two characteristic forms: one cosmological, and one kinematic. The first kind of paradox, investigated for example in the *Parmenides*, the *Timeaus*, and the *Sophist*, relates the inherence of the infinite to the topic of the unity of the cosmological All, whereby the very structure of its logos always ensures "at least one more" and thereby tends toward the ultimate destitution of the One-All in a logically/structurally implicit unlimited many. The second kind of paradox, investigated in the *Cratylus,* the *Philebus*, the *Sophist*, the *Theaetetus,* and again the *Parmenides*, is that of the thinkabilty of becoming and change, and more generally of the possibility of any thought at all of what is subject to the condition of temporal flux. These are what Deleuze has treated as the paradox of an "unlimited becoming" which threatens to show that it is impossible for anything to have any determinate identity, insofar as all such identities are situated within continua that structurally allow of indefinite increase or decrease. Both types of paradoxes, in introducing a basic structure of contradiction into the thought of the One as such, underpin late Plato's two-pronged attack on the Eleatic monism which treats being as the cosmological One-All and time and change as illusory and impossible. The development of this critique and the positive demonstration of the phenomena underlying its possibility allows Plato to rehabilitate and develop certain suggestions of Pythagorean ontology and by expounding the underlying problematic of the structural givenness of number to which it responds. As nineteenth and twentieth century commentaries have verified, he here draws just as much on the active researches of Greek mathematics into the structure of given number, including their responses to the crisis immediately

provoked by the historic discovery of the existence of incommensurable magnitudes, such as that of the diagonal of the square.[14]

There is evidence that the development of the problem of number may be closely connected with the content of what have been called Plato's "unwritten" teachings.[15] The sixth-century neoplatonist Simplicius notoriously reports descriptions by Aristotle and others (now lost) of a lecture given by Plato on the Good: in the lecture, Plato is said to have taught that the principles of all things, including the Ideas, are the "Indefinite Dyad, which is called Great and Small" and Unity.[16] There is a suggestion in Simplicius's quotations of Poryphry and Alexander that Plato had held that Unity and the Indefinite Dyad are also the elements of numbers and that each of the numbers participates in these two principles.[17] The lecture on the Good is said by Aristoxenus to have confounded Plato's listeners, who expected a lecture on ethics but were instead treated to a discussion of numbers and geometry, leading up to the claim that the Good is to be identified with Unity.[18] Beyond these second-, third-, or fourth-hand reports, there are many suggestions in Aristotle's *corpus* of the late Plato's views about the connection of forms, numbers, and the principles of unity and the "indefinite dyad" or the "great or small". Aristotle says in several places that Plato identified forms with numbers.[19] He also makes the suggestions that Plato identifies Unity with the Good (and perhaps that he identifies the Great and the Small, by contrast, with evil), and that Plato treats the "Great and Small" as matter with respect to which the One is form.[20]

In a helpful analysis, Sayre has argued that the content of the so-called "unwritten teachings" that link the problems of number with those of the structure of forms and the Good can be largely recovered from Plato's middle and late dialogues themselves, thereby illuminating Plato's final conception of the method of the dialectic and of the nature of forms and participation. It is thus not necessary, Sayre argues, to speculate about the esoteric content of the Platonic teachings alluded to by Aristotle, since they can be shown to be actually present in the late dialogues themselves. In particular, Sayre reconstructs Aristotle's statements as clearly attributing five distinct claims about forms, sensible objects, numbers, and the Great and the Small; among these are the claims that sensible objects are constituted of forms *and* the Great and the Small, and that forms are *themselves* composed of the Great and the Small and Unity.[21] As Sayre notes, while the claim that the forms are the principles or causes of sensible things is familiar from many of Plato's dialogues and is present as early as the *Phaedo*, the suggestion of a composition of the forms themselves by more basic principles would be, if it can be attributed to him, a significantly novel element of the late Plato's final thinking about them. Sayre sees this late conception as developed both thematically and methodologically in Plato's descriptions of the

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[14] Stenzel; Becker; Klein; Sayre.
[15] Aristotle refers to Plato's "so-called unwritten teachings" at *Physics* 209b14-15
[16] Sayre p. 76
[17] Sayre, p. 77.
[18] Sayre, p. 77.
[19] E.g. Metaphysics 991b9, De Anima 404b24.
[20] Metaphysics 988a7-17; Metaphysics 1091b13-14; Physics 187a17.
[21] Sayre, p. 161.

method of dialectic in the Sophist, the Statesman, and especially the Philebus, where at 16c-e, where Socrates describes a "god-given" method for pursuing problems of the one and the many generally, including (it appears) with respect to the distinctive unity exhibited by forms:

> Socrates: It is a gift of the gods to men, or so it seems to me, hurled down from heaven by some Prometheus along with a most dazzling fire. And the people of old, superior to us and living in closer proximity to the gods, have bequeathed to us this tale, that whatever is said to be [*ton aei legomenon einai*][22] consists of one and many, having in its nature limit and unlimitedness [*peras de kai apeirian en autois zumphuton echonton*]. Since this is the structure of things, we have to assume that there is in each case always one form for every one of them, and we must search for it, as we will indeed find it there. And once we have grasped it, we must look for two, as the case would have it, or if not, for three or some other number. For we must not grant the form of the unlimited to the plurality before we know the exact number of every plurality that lies between the unlimited and the one. Only then is it permitted to release each kind of unity into the unlimited and let it go. (16c-e)

On Sayre's reading, the passage is meant to formulate a methodological response to the question of how the kind of unity (monadas) that a form is can characterize indefinitely many changing particulars, without thereby becoming dispersed among them and losing its unity. The problem is a specification of the more general question of how the properties and characteristics of individuals are thinkable at all, given that they are subject to ceaseless change in time. Thus specified, the problem does not simply involve the unity of forms as such, over against sensible beings thought as completely undifferentiated or irreducibly multiple; rather, since it is also the question of how sensible things are themselves thinkable as enduring unities despite the unlimitedness of their possible change, its solution involves a unified accounting for the unity of both. Since sensory objects would, if (somehow) deprived of the relationship to Forms that allow them to be thought as distinct individuals having definite characteristics, also have no definite character and in this sense be indistinguishable from the *apeiron*, the problem is that of characterizing how determinate forms are themselves defined and gain application to the changing particulars. (p. 124) The elements of a solution to this are to be found, Sayre suggests, in the *Philebus'* development of cases (17a-e) in which a number of specific characteristics are distinguished out of a continuum of possible variation, such as the identification of particular letters from the continuum of vocables, or the identification of discrete musical notes from the continuum of sound. (p. 126) In this way, a particular discrete number of intermediate forms are introduced between the general and continuous form (for instance sound itself) and the specific instances, for which the intermediate forms then serve as measures (pp. 125-126).

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[22] As Sayre notes (p. 292) the sense of the *aei* is here ambiguous, leading to the possibilities that i) Socrates may be speaking of forms, conceived as eternal existents, exclusively of sensible objects; that ii) he may be speaking of what are said to exist "from time to time", and thus of sensible objects exclusively of forms; or that iii) both are intended. Sayre prefers the third alternative, since Plato often uses the formulation "there is such a thing is…" even in cases not specifically relating to forms, although there are also many precedents in Plato for the discussion of forms as causes.

As Sayre suggests, the methodology may be considered a further development of the method of the collection or division (or synthesis and diaeresis) proposed in the *Statesman* and the *Sophist*. As is suggested there, the key methodological idea is that the definition of a thing begins by collecting a number of instances of the kind to be defined with a view to discerning the general form they have in common, and then that form, once found, is further articulated or qualified by a repeated *diaeresis* or division of its several components, until a unique set of specific characteristics is identified that distinguish the particular kind of thing in question from others similar to it. As Sayre notes, however, the major and glaring difference between the description of the "god-given" method in the *Philebus* and the descriptions of the dialectician's art in the Sophist and the Statesman is that the latter two involve no mention of the *apeiron* or of the need to distinguish among indefinitely many single things or to articulate what is in itself a continuum having the character of the "unlimited" in the sense of indefiniteness. Sayre sees the account given in the *Philebus* as responding to a problem about unity and the apeiron – both in the sense of the "indefinitely many" and that of the indefinitely continuous -- that is already posed in the *Parmenides* (157b-158b). The idea of a unified collection of individual members, or a whole composed of parts, involves both that there is a sense of unity characteristic of the collection as a whole and that there is a sense of unity characteristic of each member as a unique individual; unity in both senses must be imposed on what is in itself non-unified in order to produce the determinate structure of whole and part. (p. 64). The possibility of identifying an individual as part of such a collection must thus result from the combination of a principle of Unity, in both senses, with a contrasting principle of the indefinitely many or multitudinous, what Plato calls in the *Parmenides* the *apeiron plethos* and which, Sayre suggests, can also be identified with the (later) mentions of the "indefinite dyad" (aoristos duas) or the "Great and the Small" of which Aristotle speaks.

On this basis, Sayre can argue that the final *Philebus* account of forms and participation involves a twofold application of the imposition of Unity on the Great and Small: first, in order to produce the determinate forms themselves, and second, in the imposition of the forms thus produced, now functioning as "measures", on the Great and Small again to produce the characteristics of particular sensible objects (p. 180). If this is right, both the Forms *and* sensible things are composed from the two principles, although according to different modes of combination. This suggestion of a unitary principle of genesis ultimately underlying both the forms and their sensory participants allows Sayre to contest both of two conflicting readings of the role of the *peras* and the *apeiron* in the *Philebus*. On the first of these, the relationship between limit and the unlimited is analogous to or anticipatory of Aristotle's account of form and matter; here, the unlimited is accordingly said to be a kind of undetermined potentiality of objects to acquire certain properties (p. 137). On the second existing view, the "unlimited" is not attributed directly to objects at all, but is rather a set of concepts which admit of variation as less or more. (p. 139). Sayre argues that both views have internal problems: the first, for example, has difficulty explaining why the imposition of Unity should produce particular objects that are in some sense valued as ordered as opposed to bad or disordered elements corresponding to other points on the same continuum; but the second has difficulty explaining how the mixing of Unity and the Indefinite could produce determinate individuals and not simply determinate *types*. Both existing alternatives, Sayre argues, are furthermore difficult to square with the text. A better alternative is to construe the combination of Unity with the *apeiron* as having the twofold application, both to the

generation of forms and, once again, to the specification of particular objects, that Aristotle also suggests in his own glosses on Plato's theory of forms and numbers. In each case, the combination allows for determinate *measure* to be imposed upon what would otherwise be the *apeiron* character of what would become or change indefinitely and without limit.

Already in the *Parmenides* (143a-144c) Plato suggests (in the voice of Parmenides) a basis for the derivation of unlimitedly many whole numbers from the One and the basic consideration that in considering its being, we already consider something that is different from it; hence the two of the One and Being, or the three of the One, Being, and difference. If we may consider the account of determinate measure given in the *Philebus* also to suggest views about the generation, not only of *whole* numbers, but of numbers generally (considered here as essentially cognate to "measures") as fixed, definite magnitudes, we can bring (Sayres argues) it into line with what Aristotle says about Plato's views about forms, numbers, and the *apeiron* (or the Great and Small). In particular, Sayre suggests that in developing the idea of a generation of determinate measures from the principles of the unlimited and unity (or limit) in the Parmenides and the *Philebus*, Plato has in mind also a general method of identifying arbitrarily rational *or* irrational magnitudes which is analogous to or actually derived from a method developed by Eudoxos and later applied in book V of Euclid's Elements, where Eudoxos is said to have been "Plato's teacher." (p. 69).

The method is essentially one of approximating an (indifferently) rational or irrational magnitude by the continued development of series of fractions. Though it is likely that the original presentation of the method was in a geometric rather than arithmetic form, it is also quite possible, Sayre argues, that some version of its arithmetic development was also known to the mathematicians of Plato's time. At that time, it would have been seen as a powerful tool of classification and comprehension in the face of the problematic discovery of irrational magnitudes; and it is clearly significant in connection with this that the main interlocutor of the *Sophist* and the *Theaetetus* is the mathematician Theaetetus, who historically contributed to the initial project of classifying irrational magnitudes and thus to the background of Euclid's book V. Sayre further notes (p. 74) that Dedekind himself, in discussing his own method for defining arbitrary real numbers as "cuts" in the rationals, cites Eudoxos's method as a direct anticipation of it. If this mathematical methodology is indeed something that Plato has, more or less explicitly, in mind with his account of the production of determinate number as well as the "measure" of fixed quantities along continua, then it yields a direct mathematical basis for the suggestion of the primacy of the principles of the limit and the unlimited in producing both forms and sensory individuals with determinately thinkable properties. And – as was undoubtedly important to Plato – if the account is indeed mathematically based in Eudoxos's method, it holds up generally even in the face of the challenge to rational thought that is *prima facie* involved in the existence of the incommensurable.

As Sayre notes, there is good evidence that these ideas about measure and number are intimately linked in Plato's thought with questions about time and becoming. The general problem of the determination of fixed points or measures within open continua gains its relevance from the consideration (which Plato may have developed, according to Aristotle's testimony, from Heraclitus) that sensory objects are generally subject to flux and change, and it is thus not evident how they can be thought as having

determinate properties at all. Within the general problem thereby posed of the relationship of generation and becoming to being in itself as thinkable, the problem of the structure of time itself takes on a particular significance, and (as we have already seen in relation to Aristotle) the question of the relation of continuity and discontinuity involved in the possibility of its being measured at determinate instants becomes particularly urgent. At *Parmenides* 156c-157b, after discussing the apparent paradox that the One, if it partakes of time, must be simultaneously becoming older and younger than itself at all times, Parmenides introduces the problem that the One, in going from being in motion to being at rest, must apparently pass through an instant at which it is neither in motion nor in rest; but there can be no such time. Thus, the "queer thing" that the instant [*to exaiphnes*] is seems to "lurk between motion and rest" (156d) and exist in paradoxical fashion between the two opposed states which something is in before and after it. By the same argument:

> Whenever the one changes from being to ceasing-to-be, or from not-being to coming-to-be, isn't it then between certain states of motion and rest? And then it neither is nor is not, and neither comes to be nor ceases to be?" -- "It seems so, at any rate." -- "Indeed, according to the same argument, when it goes from one to many and from many to one, it is neither one nor many, and neither separates nor combines. And when it goes from like to unlike and from unlike to like, it is neither like nor unlike, nor is it being made like or unlike. And when it goes from small to large and to equal and vice versa, it is neither small nor large nor equal; nor would it be increasing or decreasing or being made equal." -- It seems not." (157a-b)

As Sayre notes, the argument is general, applying not only to "the one" but to *any* particular thing, considered as a unity, as well as to any change that involves going from being in a determinate state to not being in that state. If any such change is considered as continuous, there will necessarily be a temporal moment at which the thing is neither in the state nor not in it. Thus considered, the instant is something with a paradoxical nature (phusis atopos) which seems itself to occupy "no time at all". (en chrono oudeni ousa) (156e1; p. 72).

The paradox of the instant that is here demonstrated is none other than one of the several aspects of the paradoxical nature of the "now" as a *part* of time that, as we have seen above, Aristotle points out in the *Physics*. As we saw in section 1, above, Aristotle is able to resolve or foreclose these paradoxes only insofar as he can treat the "now" not as an actual part of time but only as a limit, to be defined in the actual measurement of a span but not as a really existing part of the continuity of a continuous motion (or temporal span) prior to the measurement. Henceforth, the measured "now" will be opposed to the continuity of time as accident is opposed to essence, and the guiding conception of the *dunami on* will come to govern the whole framework in which the relationship of the temporal *apeiron* to its instantaneous determination is thinkable. This conception of the nature of the instant which renders it derivative of, and essentially dependent on, the possibility of measuring motion is also what produces, as we have seen above, Aristotle's "official" definition of time as the measured "number of motion with respect to before and after." With, however, the suggestion of a rather different basis for number in the sense of measure and its application to the determinacy of forms *and* particulars that Sayre reconstructs, we are now in a position to see in Plato's late view of the dialectic the basis for a

conception of the relationship of the infinite to time that is quite opposed to Aristotle's own. Here, in particular, and as we have seen, as well, in relation to Lautman's reconstruction of the "dialectical" conception of ideal genesis, the kind of determinacy that number in itself has is not conceived as prior to the measurement of continuous time, but rather as determined *in the same way* and *by the same principles* that make possible the measurement of sensory objects themselves – namely, that is, by the combination of the principles of the *apeiron* (or indefinite dyad) and unity or the one. Measured time is thus, here, not the *numbered* number (or the counted number), but is rather (in terms of the generative structure of its constitution) simply number, and is thereby in an original relationship with the *apeiron* and the *peras* as such. The problems of the determination (and hence the possible givenness) of time are thus not conceived as distinct from the general problems of the generation of numbers and forms, and both maintain, in the theory of their ideal genesis, an irreducible and necessarily paradoxical temporal referent. As a result, the originally paradoxical character of the *apeiron*, both in relation to the cosmological totality of time as the *aei* and to its locally continuous character, is here allowed to maintain itself to a certain extent and is preserved in the dialectical relationships that connect it to the other organizing principle of the One or unity, rather than being foreclosed or deferred, as in Aristotle's account. This has certain consequences, as we shall see, for the integrity as well as the ultimate overcoming of the forms in which Plato suggests a solution to the original problem of the mutual relationships of being, becoming, and time.

In particular, with these differences in view, it is now finally possible to return to the *logical* form in which the Eleatic Visitor proposes, in the *Sophist*, to solve the problem of the thinkability of change, becoming, and time. Recall (chapter 1 above) that in the context of the "battle of gods and giants" between those who hold that there are unchanging forms and those that hold that there are only temporally changing bodies, the Visitor extracts, from the consideration that change and motion must take part, in some way, in being, the suggestion of the peculiar structure of the *dunamis koinonia* or limited mixing among great types. The suggestion is developed in close connection with reflection on the structure of the *logos* and will yield, in its extended development, a purported solution to the problem of the sophist's ability to speak the false and thereby to represent an actual figure of the illusory. As we also saw, the Visitor's suggestion is developed along with a determinative questioning with respect to number which also yields various aspects of the attack on Parmenides that he effectively undertakes: here, for instance, the problem of the paradoxical status of the thinking or saying of the One, whereby it is already more than One, is repeated from the *Parmenides*, and seen as pointing to the irreducible plurality or manifoldness of the *logos* structure itself. The determination of the five great types and the particular kind of unity they have as a structure might also be thought to witness a determinative role of number in generating the structure of the logical koinonia. It thereby becomes possible, as well, to consider whether the *genetic* structure of number, and indeed its production out of unity and the *apeiron*, may also be considered to play a determinative role in producing the more general structural form of the logical *koinonia*, with which the Visitor attempts to solve both the problems of the thought of becoming and of non-being in themselves.

One commentator who suggests such a determinative role for the structure of *arithmos* in determining the Visitor's solutions to the problems taken up in the *Sophist* is Jacob Klein. In his remarkable study

Greek Mathematical Thought and the Origin of Algebra, Klein places Plato's conception of number in the context of what he treats as the broader Greek *arithmos* concept as developed in different but related ways by the Pythagoreans prior to Plato and Aristotle and certain neo-Platonists after him. According to Klein, in all of these Greek developments the *arithmos* never means anything other than "a definite number of definite objects" (p. 7); the slow and difficult development of a theoretical (as opposed to practical) arithmetic thus involves the gradual understanding that it is possible to make use in any counting whatsoever of prior "counting-numbers" which then appear as a kind of "undifferentiated" objects consisting in assemblages of "pure" units. The more or less complete realization of this in Plato allows him, according to Klein, to find in the structure of the *arithmos* concept "the possibility of a fundamental solution of the problem of participation (*methexis*) to which his 'dialectic' necessarily leads without, however, being of itself able to provide a solution"; in particular, Plato is able on this basis to perform a kind of repetition of the Pythagorean attempt at ordering all beings according to number, this time "within the realm of the ideas themselves." (p. 8). This Platonic conception of numbers, which finally renders them basically "separate" from the objects of sense perception, is then attacked by Aristotle (in articulating a series of criticisms which Klein finds basically convincing) as actually possible only on the basis of a prior abstractive separation in thought; the "pure" units are thereby shown to "have no being of their own" and thus to be inadequate for the foundation of an actual mathematical science.

In developing the structure of the particular relationship between dialectic and the structure of the *arithmos* that he sees as suggested in the *Sophist*, Klein relies on the prior analyses carried out by Stenzel and "especially" by Becker into the constitution of ideal numbers through the iterated diaeresis which allows for a definite number to be formed as a collection of "monads". (pp. 61-62). In ordinary activities of comparing objects and distinguishing the respects in which one object can be referred to, thought (in the sense of *dianoia*) comes into the position of being able to *count* (p. 77) several objects as of the same group, thereby discovering the relevant identity of the "unit" for counting things of *that* type or grouping. The most important recognition upon which Plato relies in developing from this the specific discovery of the logical/ontological *koinon* of the *Sophist* is, according to Klein, the discovery of a *different* kind of *koinon* characteristic of the *arithmos* as such. The conceptual basis of this discovery of the peculiar *koinon* of the arithmos is discussed, according to Klein, in the Hippias Major (300a-302b). There, in particular, Socrates notes the peculiar fact that although in general a property that is attributed to several things holds separately of each of them, in the case of number this is not so: thus Hippias and Socrates *together* are two, but neither of them individually is. Speaking more generally, the kind of *koinon* that is characteristic of predications of number is quite different than that exhibited by predications like that of "beauty" or "justice"; in the case of number but not in the other cases, what is predicated of a number of things "belongs to these things only in respect to their community, while each single thing taken by itself is one." (p. 81) Klein sees this as being deeply related to the problems of the Platonic *methexis*: in particular, the problem to which it responds is "nothing less than the *aporia*, the quandary, of the *Parmenides* (130e-131e) and of the *Philebus* (15b4-8) – namely how it is possible that one idea in its unity and wholeness is "distributed over" the many things which "partake" in it." (p. 80). This is just, according to Klein, the general problem of the *methexis*, albeit "merely on the level of the

dianoia"; the problem, however, "reaches its full sharpness and force only when the relation of an idea of a higher order to the ideas under it, of a 'genus' to its 'species', is concerned." (p. 80)

Having distinguished the two types of *koinonia* and suggested applying the difference to the consideration of the relations of ideas with one another, Klein suggests that the *Sophist* proposes to resolve a "fundamental Platonic problem," that of the "community of ideas" (p. 82) or of the possibility of ideas or types mixing or refusing to mix with one another. According to Klein, the solution in terms of the particular *koinonia* of number and in accordance with the ideal genesis of number from the *aoristos duas* is crucially important in underlying the account the Visitor gives of the sophist's image-making and the character of the image in general. In particular, if the phenomena of imitation, mirroring, similarity, *etc.* are to be possible at all, it must be because being has a "primal character of *"imageability,"*; this character arises, according to Klein, from a kind of inherent underlying principle of doubling or of the "twofold in general" which is nothing other than the *aoristos duas*. (p. 82). In this way, according to Klein, the *aoristos duas*, which is the "*arche* of all duality and thus of all multiplicity" finds perfect embodiment in the person of the sophist. It is on this basis, according to Klein, that the Visitor is also able to articulate the specific structure which makes the *me on* a possible object of study (p. 85) and thus confirms the possible relation of non-being with being which occurs in the person of the sophist. The structural relationship which first allows both kinesis and stasis to figure in being – though neither can mix "at all" with the other – points to the more articulated *koinon* structure of the great types. According to Klein, this structure must be characterized by the special arithmetic *koinon*, rather than the *koinon* of generality more usually characteristic of *eide*, in order for the *gene* to subsume the several ("(fintely) many", Klein says) *eide* that fall under them without losing their own unity. The relationships involved in the possible but limited mixing of types point to characteristics that the types have not individual but only "in community", and thus can only be structured in terms of the specifically arithmetic *koinon*. (p. 90). In this way, according to Klein, the special structure of the *arithmos* and the specific kind of *koinon* it involves is crucial to the possibility of presenting the structured *logical* relationships among ideas and types of subsumption and articulation that the late dialectical method of synthesis and diaeresis is charged with demonstrating.

It is certainly possible to see the structure of the "great types" and the methodology of synthesis and diaeresis on which their discernment is based, in the *Sophist*, as "on the way" to a taxonomy of species and genera of roughly an Aristotelian kind. In particular, both the relationships of mixing and failing to mix among great types and their individual relationships with the less elevated *eide* suggest what would today be treated as logical relationships of mutual exclusion, partial overlap, or greater or less generality, and Plato, insofar as he endorses the Visitor's solution, might thus be seen as presenting with the structure of the great types the most general or overarching framework for considering such logical or conceptual relationships generally, a framework to be filled in in more detail later by the synthetic/diaeretic determination of the more specific *eide*. Such a taxonomic structure would itself depend on the possibility of articulate connection, with differentiation, within the unity of a higher *genus* that appears to be crucial to the analogy or identity that Plato draws between the structure of ideas and numbers. Thus Klein, relying on Aristotle, is able to suggest near the conclusion of his discussion of Plato (p. 98) that "the doctrine of the *gene* as eidetic numbers must, finally, also furnish

the foundation of an *eidetic logistic*" that articulates conceptual realtions "by means of *analogia*," or "proportion": this is, according to Klein, the structural basis for the discussion of the more articulate relationships between ideas such as that of *sophrosone* and *dikaiosone* in the *Republic*, or for the "*taxis* of elemental materials in the *Timeaus*". (p. 98)

Given the "special" *koinon* of number and its functioning to ensure a special kind of limited conceptual relationship among ideas and types, it is thus possible to relate the general idea of a generation of number from unity and the *aoristos duas* to the possibility of a specifically logical taxonomy of types or categories, presided over by the great *gene* articulated in the *Sophist*. Of course, there are problems with aligning this structure in detail with any suggested by Aristotle; for instance, Aristotle does not understand being itself as a most general type or category (but rather in terms of the enigmatic *pros hen*), and the role of difference in Aristotle's categorical structures is not that of a "great type" which would permeate all others. But in view of the deeper underlying structure that appears to be at the basis of number itself for Plato – that of the combination or application of the principles of Unity and the *aoristos duas* – it is worth asking whether a general and total structure of categories can indeed be founded in this way without involving or invoking, at the same time, an irreducible structure of paradox which subsequently characterizes the structure of generality involved in the application of any logical structure of maximal generality at all to a world of beings in time. One may suspect not, indeed, given the way that the invocation of the great types in the Sophist also aims to respond to the *temporal* problem of thought and being also involves at a fundamental level the problem of given time and its relationship to the original structure of number.

In particular: is there, in fact, a specific *koinon* of the *arithmos* of the kind Klein suggests? This is in fact doubtful, in view of the radical nineteenth and twentieth-century development of the constitutive ideas of number, logical structure, and the infinite. In particular: the profound observation that Socrates makes in the *Hippias Major* of the possibility of a kind of predication that holds of a group of several individuals without holding of them individually is nothing other than one of the key observations on which Frege bases his argument, in the *Foundations of Arithmetic*, that a judgment of number is not *in any sense* a judgment of the properties of individuals, but rather a judgment about *a concept*. (see especially sections 45-46 of the *Foundations*). In the late (1919) text "Notes for Ludwig Darmstaedter", Frege himself cites what Plato recognizes as the peculiar character of number judgments in the *Hippias Major* as a basic determinant of his own thinking:

> I started out from mathematics. The most pressing need, it seemed to me, was to provide this science with a better foundation. I soon realized that number is not a heap, a series of things, nor a property of a heap either, but that in stating a number which we have arrived at as the result of counting we are making a statement about a concept. (Plato, *Hippias Major*.) (p. 362)[23]

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[23] I am indebted to John Bova for pointing out to me the reference, as well as for helpful discussions about the conceptual connections here.

It is a profound consequence of this recognition, as it is developed in the *Foundations,* that being "one" or "unity" is *not* a property or predicate of objects (sections 29-33).   As a consequence, it is also not possible (section 45) to consider numbers to be composed of (a number of) pure and identical "units".[24]

Basing himself directly on the specific observation already made by Plato in the *Hippias Major*, Frege can thus argue that neither any particular number, nor the structure of numbers in general, is in any sense either a *collection* of individual objects or a property of objects so collected, *even* "in community." There is thus, according to Frege's argument, no *koinon* of number in general, and it is (accordingly) not possible to refer the "logical" structure relating *eide* and *gene* "in community" to such a structure, as Klein attempts to do.[25]   Numbers are rather, on Frege's argument, individual and quite distinct objects of thought: although the judgment of number is in each case the judgment of a feature of a concept, this does not mean that numbers are in any sense subjective or irreal.   Rather, the possibility of judgments of number is given (in accordance with what has been called 'Hume's principle') along with the possibility of judgments of equinumerosity in general.   And this possibility is not based on properties of any specific objects or of objects in general, but is rather co-given in an original and objective way along with the givenness of objects to thought itself.

It is therefore necessary, if we accept Frege's argument, to see number and the possibility of counting as *originally* based, not in a specific *koinon* (or any other) structure of ideas or particular logical relations, but in the overall character of the way that objects and phenomena are themselves and *in general* given to be thought.   As we have already suggested (chapters 1 and 4 above), to see the structure of number in this way is not to see it as either the outcome or basis of a specific structure of relations among beings of *any* kind or type, no matter how "superior" or "universal", but rather as co-implying something like the character of "being itself" in the way that it determines the *possible* givenness of beings.   As we have seen repeatedly (esp. chapters 1 and 4), this indication of the structure of possible givenness always unfolds in determining and determined relationship with the problematic constitutive dynamics of the ideas of unity, the finite, the infinite and the total.   None of these ideas operate or can operate as predicates with respect to beings individually or collectively, but their dialectical relationships articulate the very underlying structure, indifferently "metalogical" or "ontological" that is indicated in the reflection that considers the being of beings in the sense of their *possible* givenness to thought.

As we have already seen (chapters 4 and 6), another set of consequences of Frege's understanding of number that is relevant to the shape of this overall structure is that involved in Russell's paradox and the

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[24] "If the things to be numbered are called units, then the unconditional assertion that units are identical [gleich] is false.   That they are identical in certain respects is no doubt correct but worthless.   The difference between the things to be numbered is actually necessary if the number is to be greater than 1." (p. 98)

[25] Of course, none of this establishes that Plato *himself,* either at the time of the *Sophist* or at any time, did not think of things along the lines that Klein suggests.   Additionally, it should be noted that Klein is, throughout his analysis, at pains to distinguish the Greek conception of number and arithmetic from our modern one, and so would probably on those grounds resist the application of Fregean arguments here.   Nevertheless, whatever Plato himself might have thought, there is a legitimate question about the real form of the basis of number and counting that is posed here, and which respect to which it is possible to pursue an answer that would also reflect what we now know about the various problems and structural relationships of unity, number, and the infinite.

other set-theoretical and semantic paradoxes of totality and reflexivity formally related to it. In particular, it is a consequence of Frege's view of number as initially developed that concepts must be able to determine their extensions in general; but this assumption cannot be maintained along with reference to the totality of all concepts while avoiding contradiction. The result figures, as well, in the ultimate consequences of Cantor's development of the determinate possibility of coherently thinking infinite totalities as such by means of set theory. The axiomatic structures that subsequently formulate set theory in such a way as to preclude the possibility of contradictions of this type do so by effectively prohibiting the possible formation of, or reference to, the *universe* as a whole: in this way they prohibit or preclude the traditional "One-All" of maximal infinite totality. But what can be glimpsed here, and is unfolded in determinate form in the constitutive metalogical problematics that transformed first set theory and then the theory of proof in the first decades of the twentieth century, is the insistence of the superior dialectic of metalogical ideas and the basic paradoxes they articulate in relation to the question of the possible thought of beings in general.

It is the inquiry into this structure that leads Frege to deny the suggestion of a distinct *koinon* of number in particular or in general as any sort of grouping of individuals into "ones". It is possible to see the late Plato, especially insofar as he ultimately adopts an accounting for the givenness of *all* beings (sensible ones as well as ideas) to thought in terms of the ultimate *principles* of unity and the *unlimited many*, as engaged in a structural similar inquiry, and indeed one with similarly aporeatic ultimate results. But to see his inquiry in this way, we must distinguish it from the Visitor's specific *solution* to the problems of becoming and falsehood that is offered in the *Sophist*. This is not only because we may suspect that Plato uses the Visistor as a mouthpiece for views that are not his own or not fully endorsed by him at the time, but also because (as Sayre also notes), the Visitor's account of the method of synthesis and diaeresis is not the same as the one that Socrates later endorses in the *Philebus*. Most obviously, it does not (or does not yet) involve the particular dynamics of the *apeiron* in relation to limit that is crucial to the structure of the "god-given" method there; with respect to this later development, both the method the Visitor recommends and the particular "solution" it yields must be considered, at best, only partially successful. In particular, as we have seen (chapter 1 above), the solution in terms of the *koinonia* of limited mixing between types presupposes the *simultaneously* logical, ontological, and psychological parallel givenness of a structure that it itself cannot ultimately explain. The *simultaneity* of the orders in which the properties of beings *and* their possible thinking – *including* the thought of their non-being – take place is here crucial, and its assumption (as we have seen in connection, also, with Aristotle), amounts to something like the assumption of a logical-ontic construal of thinkable being in the temporal form of the present as such. It is also to be noted here that nothing in the Visitor's official solution even so much as responds to the problems of the relationship of continuity and discontinuity, such as they are involved in the form of the moment or "now", as Plato's later development of the method in terms of the *apeiron* as the indistinct at least attempts to do. The Visitor's account of the co-existence of change and being, as well as his account of non-being and falsehood, must then be seen as essentially presupposing this ambiguously simultaneous logical, ontological, and psychological *koinonia* as a simply given ontic structure of co-presence, without actually penetrating to the deeper ontological ground of *its* possible givenness. This deeper ground must be the underlying structure of given time, as it is

articulated and undermined in the constitutive dynamics linking the ideas of unity, number, and the infinite.

As we have seen (chapter 4), it is only this failure to pose and pursue the ultimately ontological (or, metalogical) questions here that allows the Visitor to portray non-being and the possibility of illusion in general as the result of the limited "mixing" of difference with other *eide* or *gene*, thus grounding it in what must then seem to be a logically *regulated* structure of combination. From the perspective of the later development of the specific problematic structure of the *apeiron* (which is, however, already fully visible in relation to the paradoxes of the one and the others in the *Parmenides*), this is visibly an attempt to limit or modify the capacity of difference to subvert and transform fixed identities, a capacity which is only fully brought out in the specifically "unlimited" structure of the *aoristos duas* itself. In chapter 4, above, we saw reason to suspect, on the basis of the development of the problems of the original structure of negation, non-being, and contradiction, that the specific structure of non-being is ultimately not to be referred to difference as a form or type, but rather to a prior differentiation that is anterior to all given beings and insists on the level of the possible givenness of the whole. Insisting in this way, it communicates irreduciblty with the constitutive ideas of finitude and the infinite as well. From this perspective, that neither the *aoristos duas* nor unity are, in Plato's most developed thought, ideas, but rather superior principles of the genesis of ideas *and* sensory objects, both in their being *and* their becoming, means that the dialectic of the determination of the being of beings is here referred, finally, not to beings but to the superior principles that are, in governing their possible disclosure, also governing with respect to the givenness of numbers as such. But they do not do so without *also* witnessing the insistence of an original structure of paradox at the metalogical/ontological basis of this co-givenness itself, which is clarified and confirmed in our time by the train of implications following from Cantor's radical development of the constitutive ideas of the one, many, limit and unlimited.

If this is correct, then it also provides obvious grounds for questioning the ultimate groundedness of a taxonomic/categorical structure of being of the Aristotelian kind. The suggestion here is not that it is not possible to define particular regional ontologies or typologies of beings according to their specific properties, but rather that the development and imposition of any such structure depends upon a more basic structural dynamic of totality and identity which is both presupposed in any such taxonomy and shows that no such system can provide a non-contradictory and unique ordering of beings as a whole. Viewed another way, this is because the way that categories fix beings in their structure depends upon a basic disavowal of the problems of their *temporal* givenness to thought, the specific flattening or levelling of the underlying form of temporality that (Heidegger suggests) is also the disavowal of the temporally based *existentiell* structure at the actual basis of the constitution of all categories and of the "theoretical" attitude in which it takes place. We have witnessed, in the investigations of the last several chapters, how this original temporal structure can also be related to that of the possible disclosure of entities, the prior "ontology" of their possible truth.   To grasp the way in which the problems of the structure of the infinite are here involved is to see, also, some of the constitutive structural moments of a "more original" temporality that precedes but also undermines the "logical" or taxonomic fixation of beings in the categories that relate and structure them as a whole.

It is also possible, with this in mind, to return to the specific question of the status of contradiction and the specific (regulative or constitutive) force of the "law of non-contradiction" itself. Notoriously, it is difficult to say what the law of non-contradiction is actually *about*: it can, and has, been characterized alternatively (among other things) as an ontic principle governing entities and their possible properties, as a psychological principle governing actual thought or the attribution of it, as a normative principle governing *possible* contents of thought, or as a "logical" principle governing inferential or deductive relations within a general formal logic. There is also, as we have seen (chapters 4 and 6 above) a very basic ambiguity about the relationship of the principle to time: is the principle to be formulated, as Aristotle does, in a way that specifies what it prohibits as a co-existence in time (i.e. that it is impossible for a thing to have a property and not to have it *at the same time*) or should it, as a supreme principle of *analytic* judgment, have no specific relationship to time, as Kant suggests? As we have seen, Aristotle's original development of the principle as one prohibiting the co-existence of opposed properties in time is structurally dependent upon the assumption or presupposed form of simultaneity in general, the undecidably spatial-temporal structure that is invoked and exploited as the pivot of Aristotle's account of time as the number of motion under the heading of the *hama*. Time is then itself logically structured by the prohibition of contradiction, or seen as the condition under which a contradiction can be rendered non-contradictory by being distributed within it; the basis of this conception is the ambiguous and ultimately paradoxical inherence within it of the successive structure of discrete "nows".

On the other hand, the Kantian (or Fregean) intuition that makes the law of noncontradiction extra-temporally applicable to a realm of previously defined contents in general itself presupposes *their* simultaneity, a simultaneity that (as Heidegger points out) is all the more originally "temporally" determined in being determined as the co-presence of the *a priori*. From the perspective articulated by the more original structural dynamics we have suggested here, the law of contradiction, like the "phenomenon" of contradiction itself, is related to time in neither of these two ways – not as having either the "temporality" of the governance of intra-temporal objects and events *or* the regulation of "extra-temporal" relationships among contents – but rather as co-articulating the original *form* of time as given. Here, the "force" of the law of contradiction is not psychological, ontic, or "logical" but rather metalogical or (what is the same) ontological in pre-structuring and providing a basis for the specifically regulated relationship of what are subsequently conceived as beings in *any* of these domains. It also does not, thought originally and in the context of the metalogical problematics that it articulates, any longer itself have the force of governing or regulating any specific kind of beings or beings in general. It is rather to be illuminated in terms of the more original structure of the metalogical duality that links it irreducibly and undecidably to the structure of completeness, consigning thought in its relation to being to be either consistent and incomplete, or complete and inconsistent. These possibilities also then present themselves as original and undecidable possible forms of the temporality of objects and their givenness. In the first case (Aristotle's intuition) the guarantee or requirement of consistency ensures that objects and phenomena will only ever be given as incomplete, as given "up to" a finite limit or as constantly unfolding without end in what is then thought as the temporality of the dunamai on. In the second (the one developed by Plato in the *Parmenides*) the total givenness of the structure of time to thought, along with the givenness of the one, yields the original paradoxes of totality, temporality, and becoming that propose the ultimately aporeatic structure of the really inherent *now* as time's very form.

If the Visitor's attempted solution to the problems of the thinkability of becoming, change, and non-being in the *Sophist* in accordance with the method of synthesis and diaeresis invoked there can be considered only, at best, partially successful, does Plato's apparent later further development of this structure in terms of the *apeiron* and unity ultimately succeed in solving these problems in a complete and consistent way? In fact, it does not. As we have already seen, the metalogical (or ontological) problematic can here do no better than point to the originally *paradoxical* situation of the dialectic that links being and becoming, a paradoxical structure that is unfolded with the constitutive paradoxes of totality, reflexivity, givenness and time themselves. That Plato is eminently aware of these paradoxes is shown by their elaborate development in the *Parmenides*, and if he is ultimately thereby moved to refer to the more basic structure of the *apeiron* and the *one* in their problematic relation and to place this relation at the basis of the very possibility of the givenness of forms and of objects, he does not on this basis resolve these original problems themselves but rather only contributes to demonstrating their underlying structure. Even given all that Plato says, or what we can infer or guess from what he is reported to have said, about the role of the two principles of the unlimited dyad and the one in giving rise to numbers, forms, and the determinate nature of things in temporal flux, it remains possible to pose the paradox of the thinkable being of the one as such, in terms of which it will always invoke "one more," unto the infinite, and the related paradox of the unlimited possibility of differentiation which will never settle upon a determinate identity for a singular something until it can be subject to an infinite *complete* process of maximal differentiation. Above all, there remains the originally paradoxical character of the presence of the instant, which seems to take place in no time at all and to be capable of having no determinate character, but rather to be in itself the medium of the inherence of all contradictions, of the contradictory as such. The "reappearing" Socrates of the *Philebus* presents the method that he recommends there in full and apparent awareness of these structural paradoxes, and does not so much suggest that the method itself can resolve them completely and finally as that it is itself structurally prescribed by them. The "god-given" method is, in any case, appropriate as a response to the more original ontological situation "passed down" from ancients who are themselves situated "closer" to the gods, and the basis for its specific availability as a techne is attributed mythologically or metaphorically, as in the *Phaedrus* (see above, chapter 4), to the problematic methodological gift of the god Theuth to men in granting the original possibility of letters and writing. If the dialectical method is thus presented as any kind of solution to the constitutive problems of totality, infinity, and temporal becoming, these are thus presented as ontologically given problems from which, literally, only a god can save us. It remains possible, before or beyond this mythological, theological, or onto-theological reference and whatever it might be thought to guarantee in Plato's text, to witness there the insistence of the underlying problematic dynamics of paradox that are themselves unfolded again in contemporary investigations into the metalogical structure of being and time.

According to this metalogical structure as I have tried to suggest it here, the paradoxes of the infinite inhere in the structure of given time in two senses: both cosmologically, in relation to time as a whole, and punctually, in relation to the structure of the instant or "now" that is always becoming-other and always destroying itself. If we can indeed see in Plato's text an original development of these problems, one which is, as I have suggested, subsequently covered up and put out of play by the Aristotelian

conception of the *dunami on* which will regulate thought about the infinite up until Cantor, it is nevertheless possible, on the basis of contemporary metalogical as well as ontological investigations, to bring them out and clarify them today in a new and different light. Since such a clarification of the underlying problematic situation *also* has the effect of exposing to questioning, in its light, the original form in which the givenness of time is thought in the Western tradition, it also relates in a determinate way to the articulate closure of the metaphysical epoch of presence that Heidegger announces. It here becomes possible, in particular, to think the original problematic structure of given time on the basis of a dynamic of ideas that does not any longer presuppose the givenness of time in the privileged form of a (simultaneous) present, or at any rate provides basic terms for deconstructing and displacing this privilege on the basis of a more structurally basic thinking of the form of presence itself. That such a thinking becomes possible at a certain determined moment is one of the implications of what Heidegger calls *Ereignis*, and the specific historical and also metalogical conditions that make it possible *also* can suggest forms and means for a thinking of being and time that is no longer constrained within the presumptive structures of ontotheology as grounding and grounded from below and above. We turn to the more detailed consideration of the structure of this possible thinking, and what (in particular) it implies about the *contemporary* ontological situation, in the next chapter.